8. MATEMATIKA A2 FELADATSOR

1. $\mathbb{R}^3$-ben alteret alkot-e az alábbi halmaz:

   (a) $\{\begin{pmatrix} a \\ b \\ 0 \end{pmatrix} : a, b \in \mathbb{R}\}$

   (b) $\{\begin{pmatrix} a \\ b \\ 1 \end{pmatrix} : a, b \in \mathbb{R}\}$

   (c) $\{\begin{pmatrix} a \\ b \\ c \end{pmatrix} : a + b + c = 0\}$

   (d) $\{\begin{pmatrix} a \\ b \\ c \end{pmatrix} : a, b, c \text{ egészek}\}$

   (e) $\{\begin{pmatrix} a \\ b \\ c \end{pmatrix} : a, b, c \text{ között van nemnegatív és nempozitív is}\}$

2. $\mathbb{R}^{2\times 2}$-ben alkot-e az alábbi halmaz:

   (a) $\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a, b, c, d \geq 0\}$

   (b) $\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : a + b + c + d = 0\}$

   (c) $\{\begin{pmatrix} a & 0 \\ 0 & b \end{pmatrix} : a, b \in \mathbb{R}\}$

3. Bázist alkotnak-e $\mathbb{R}^2$-ben az $\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_k$ vektorok? Ha igen, akkor írja fel ebben a bázisban a $\underline{v} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ vektor koordinátáit!

   (a) $\underline{u}_1 = \begin{pmatrix} 4 \\ 5 \end{pmatrix}, \underline{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

   (b) $\underline{u}_1 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}, \underline{u}_2 = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$

   (c) $\underline{u}_1 = \begin{pmatrix} -3 \\ 2 \end{pmatrix}, \underline{u}_2 = \begin{pmatrix} 2 \\ 4 \end{pmatrix}$

   (d) $\underline{u}_1 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}, \underline{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \underline{u}_3 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$

4. Bázist alkotnak-e $\mathbb{R}^3$-ben az $\underline{u}_1, \underline{u}_2, \ldots, \underline{u}_k$ vektorok? Ha igen, akkor írja fel ebben a bázisban a $\underline{v} = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ vektor koordinátáit!

   (a) $\underline{u}_1 = \begin{pmatrix} 2 \\ 5 \\ 3 \end{pmatrix}, \underline{u}_2 = \begin{pmatrix} 4 \\ -1 \\ 3 \end{pmatrix}$

(b) $\underline{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 2 \\ 3 \\ -1 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 4 \\ 7 \\ 5 \end{pmatrix}$

(c) $\underline{u}_1 = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 7 \\ 6 \\ 5 \end{pmatrix}$

(d) $\underline{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\underline{u}_4 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

5. Milyen z esetén alkotnak bázist az alábbi vektorok?

(a) $\underline{u}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 4 \\ 7 \\ -1 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 7 \\ 2 \\ z \end{pmatrix}$

(b) $\underline{u}_1 = \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 5 \\ -1 \\ -1 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} z \\ 1 \\ 1 \end{pmatrix}$

(c) $\underline{u}_1 = \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} z \\ -11 \\ 2 \end{pmatrix}$

6. Lineárisan függetlenek-e $\mathbb{R}^3$-ben az alábbi vektorok?

(a) $\underline{u}_1 = \begin{pmatrix} 4 \\ 2 \\ -6 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$

(b) $\underline{u}_1 = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 4 \\ 2 \\ 1 \end{pmatrix}$

(c) $\underline{u}_1 = \begin{pmatrix} 1 \\ 3 \\ 5 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 2 \\ -1 \\ 3 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 1 \\ -4 \\ -2 \end{pmatrix}$

(d) $\underline{u}_1 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

(e) $\underline{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$, $\underline{u}_2 = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\underline{u}_3 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$, $\underline{u}_4 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$

7. Lineárisan függetlenek-e $\mathbb{R}^{2\times 2}$-ben az alábbi mátrixok?

(a) $\underline{\underline{M}}_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, $\underline{\underline{M}}_2 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $\underline{\underline{M}}_3 = \begin{pmatrix} 1 & 4 \\ 9 & 16 \end{pmatrix}$

(b) $\underline{\underline{M}}_1 = \begin{pmatrix} 1 & 2 \\ 3 & -1 \end{pmatrix}$, $\underline{\underline{M}}_2 = \begin{pmatrix} 4 & 7 \\ -3 & 2 \end{pmatrix}$, $\underline{\underline{M}}_3 = \begin{pmatrix} 2 & 3 \\ -9 & 4 \end{pmatrix}$

(c) $\underline{\underline{M}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\underline{\underline{M}}_2 = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$, $\underline{\underline{M}}_3 = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$ $\underline{\underline{M}}_4 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$

(d) $\underline{\underline{M}}_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $\underline{\underline{M}}_2 = \begin{pmatrix} 1 & 2 \\ 0 & 0 \end{pmatrix}$, $\underline{\underline{M}}_3 = \begin{pmatrix} 1 & 2 \\ 3 & 0 \end{pmatrix}$ $\underline{\underline{M}}_4 = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ $\underline{\underline{M}}_5 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$