3. feladatsor

3/a $\underline{a}=(2,4,1)$, $\underline{b}=(3,6,-1)$

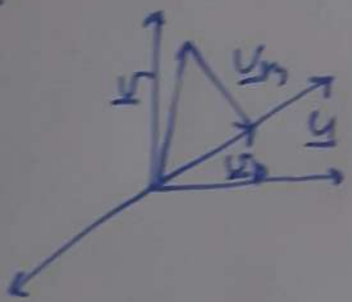
$$\underline{a}\underline{b}=|\underline{a}||\underline{b}|\cos\gamma \Rightarrow \cos\gamma=\frac{\underline{a}\underline{b}}{|\underline{a}||\underline{b}|}=\frac{6+24-1}{\sqrt{21}\sqrt{46}} \Rightarrow \gamma=21{,}08^\circ$$

4/a $\underline{a} \perp \underline{b} \Leftrightarrow \underline{a}\underline{b}=0$

$$\underline{a}=(x,3,1),\ \underline{b}=(4,7,-5) \Rightarrow \underline{a}\underline{b}=4x+21-5=0 \Rightarrow 4x=-16$$

$$x=-4$$

5/a



$$\underline{v}_p=\frac{\underline{u}\underline{v}}{|\underline{u}|^2}\underline{u}$$

$$\underline{u}=(6,2,1),\ \underline{v}=(4,1,2)$$

$$\underline{v}_p=\frac{24+2+2}{36+4+1}\underline{u}=\frac{28}{41}(6,2,1)=\left(\frac{168}{41},\frac{56}{41},\frac{28}{41}\right)$$

$$\underline{v}=\underline{v}_p+\underline{v}_n \Rightarrow \underline{v}_n=\underline{v}-\underline{v}_p=(4,1,2)-\left(\frac{168}{41},\frac{56}{41},\frac{28}{41}\right)=\left(-\frac{4}{41},-\frac{15}{41},\frac{54}{41}\right)$$

7/a

$$ter=\frac{|\underline{u}||\underline{v}|\sin\alpha}{2}=\frac{|\underline{u}\times\underline{v}|}{2}$$

$$\underline{u}=(4,-1,2),\ \underline{v}=(1,1,2)$$

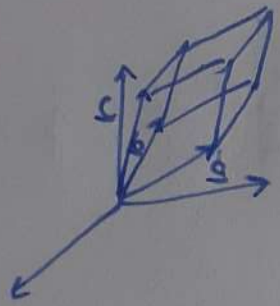
$$\underline{u}\times\underline{v}=\left|\begin{array}{ccc|cc} \underline{i} & \underline{j} & \underline{k} & \underline{i} & \underline{j} \\ 4 & -1 & 2 & 4 & -1 \\ 1 & 1 & 2 & 1 & 1 \end{array}\right| = (-2-2,\ 2-8,\ 4-(-1))$$

$$=(-4,-6,5)$$

$$\Rightarrow ter=\frac{\sqrt{16+36+25}}{2}=\frac{\sqrt{77}}{2}$$

4. feladatsor

1/b



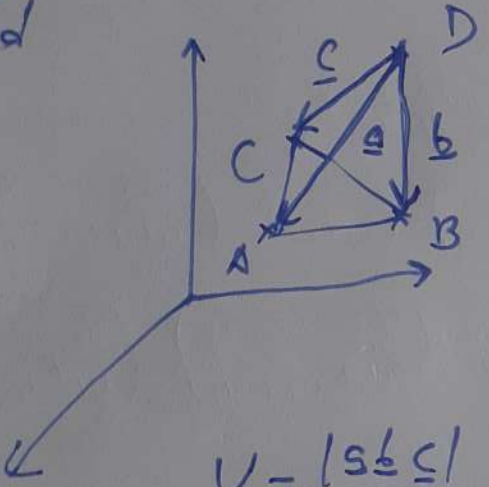
$V_{paralelepipedon} = |\underline{a}\,\underline{b}\,\underline{c}|$

$\underline{a} = (-1,-1,3),\ \underline{b} = (3,2,1),\ \underline{c} = (1,-2,3)$

$$\underline{a}\,\underline{b}\,\underline{c} = \left|\begin{array}{ccc|cc} -1 & -1 & 3 & -1 & -1 \\ 3 & 2 & 1 & 3 & 2 \\ 1 & -2 & 3 & 1 & -2 \end{array}\right| = -6-1-18-6-2-(-9) = -24$$

$\Rightarrow V = |-24| = 24$

1/d



$\underline{a} = \overrightarrow{DA} = (-2,3,-4)$

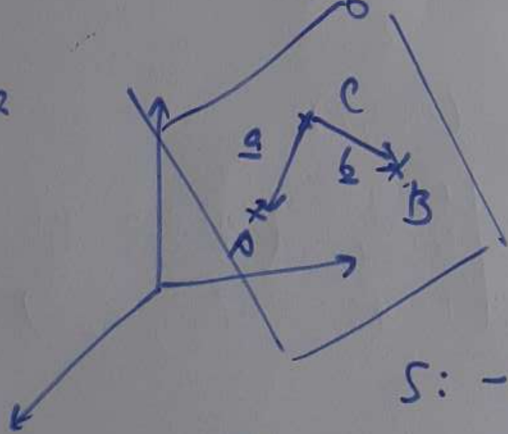
$\underline{b} = \overrightarrow{DB} = (-3,3,1)$

$\underline{c} = \overrightarrow{DC} = (-6,1,-1)$

$$\underline{a}\,\underline{b}\,\underline{c} = \left|\begin{array}{ccc|cc} -2 & 3 & -4 & -2 & 3 \\ -3 & 3 & 1 & -3 & 3 \\ -6 & 1 & -1 & -6 & 1 \end{array}\right| = 6-18+12-72-(-2)-9 = -79$$

$$V = \frac{|\underline{a}\,\underline{b}\,\underline{c}|}{6} = \frac{|-79|}{6} = \frac{79}{6}$$

3/a



$$\underline{n} = \overrightarrow{CA} \times \overrightarrow{CB} = \left|\begin{array}{ccc|cc} \underline{i} & \underline{j} & \underline{k} & \underline{i} & \underline{j} \\ 2 & 0 & 4 & 2 & 0 \\ 4 & 2 & 6 & 4 & 2 \end{array}\right| =$$

$\overrightarrow{CA} = (2,0,4)$

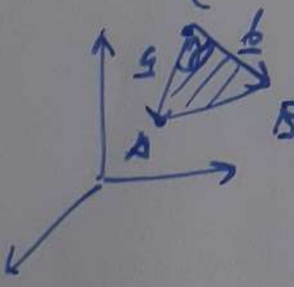
$\overrightarrow{CB} = (4,2,6)$

$(0-8, 16-12, 4-0) = (-8,4,4)$

$S: -8(x-1)+4(y-2)+4(z-3) = 0$

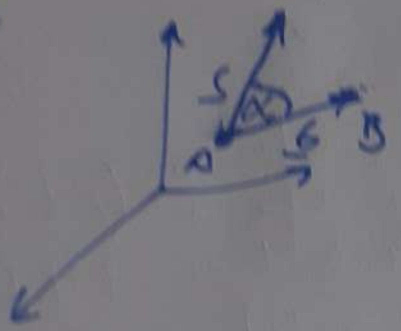
$-8x+4y+4z = 12$

3/b



$$t_{er} = \frac{|\underline{a}||\underline{b}|\sin\gamma}{2} = \frac{|\underline{a}\times\underline{b}|}{2} = \frac{\sqrt{64+16+16}}{2} = \frac{\sqrt{96}}{2} = \sqrt{24}$$

3/c



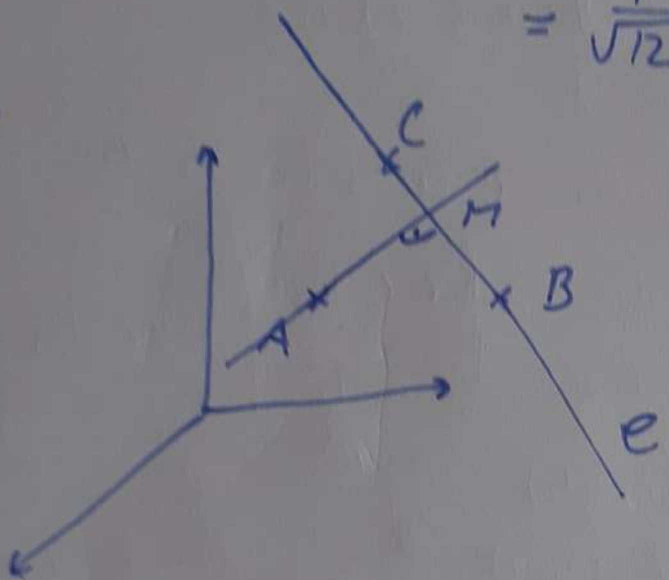
$\underline{b} = \overrightarrow{AB} = (2,2,2)$

$\underline{s} = \overrightarrow{AC} = (-2,0,-4)$

$$\underline{b}\,\underline{s} = |\underline{b}||\underline{s}|\cos\alpha \Rightarrow \cos\alpha = \frac{\underline{b}\,\underline{s}}{|\underline{b}||\underline{s}|} =$$

$$= \frac{-4+0-8}{\sqrt{12}\sqrt{20}} = \frac{-12}{\sqrt{240}} \Rightarrow \alpha = 140{,}77^\circ$$

3/f



e irányvektora: $\underline{v}_e = \overrightarrow{BC} = (-4,-2,-6)$

e paraméteres egyenletrendszere:

$$x = 3-4t$$
$$y = 4-2t \qquad t \in \mathbb{R}$$
$$z = 5-6t$$

Az A pontot tartalmazó e-re merőleges sík egyenlete:

$\underline{n}_S = \underline{v}_e = (-4,-2,-6)$

$$-4(x-1)-2(y-2)-6(z-3)=0$$

$$-4x-2y-6z=-26$$

M az S és e metszete: $-4(3-4t)-2(4-2t)-6(5-6t)=-26$

$$56t = 24$$

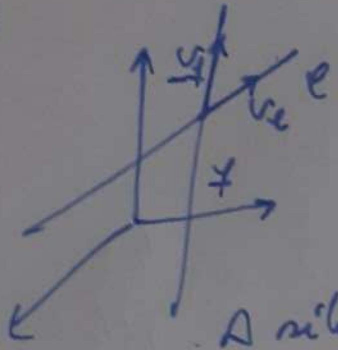
$$t = \frac{24}{56} = \frac{3}{7}$$

$$\Rightarrow M = \left(3-4\cdot\frac{3}{7},\ 4-2\cdot\frac{3}{7},\ 5-6\cdot\frac{3}{7}\right) = \left(\frac{9}{7}, \frac{22}{7}, \frac{17}{7}\right)$$

$$d = |\overrightarrow{AM}| = \sqrt{\left(\frac{2}{7}\right)^2+\left(\frac{8}{7}\right)^2+\left(-\frac{4}{7}\right)^2} = \sqrt{\frac{84}{49}}$$

$$\overrightarrow{AM} = \left(\frac{2}{7}, \frac{8}{7}, -\frac{4}{7}\right)$$

4/c



$\underline{v}_e = (2,1,-1)$

$\underline{v}_f = (1,4,2)$

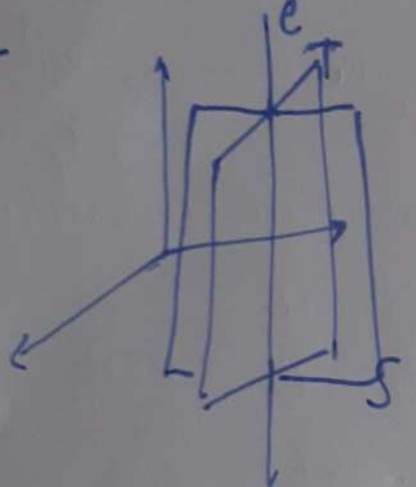
$\underline{n} = \underline{v}_e \times \underline{v}_f = \left| \begin{array}{ccccc} i & j & k & i & j \\ 2 & 1 & -1 & 2 & 1 \\ 1 & 4 & 2 & 1 & 4 \end{array} \right| = (2-(-4), -1-4, 8-1) = (6,-5,7)$

A sík egy pontja: $t=0$: $P(3,-1,2)$

Sík egyenlete: $6(x-3)-5(y-(-1))+7(z-2)=0$

$6x-5y+7z=37$

6/b



1. Megoldás: e benne van S-ben $\Rightarrow$

$$\left.\begin{array}{l} e \perp \underline{n}_S \Rightarrow \underline{v}_e \perp \underline{n}_S \\ \text{Hasonlóan: } \underline{v}_e \perp \underline{n}_T \end{array}\right\} \Rightarrow \underline{v}_e = \underline{n}_S \times \underline{n}_T$$

$\underline{n}_S = (2,3,4)$

$\underline{n}_T = (-1,4,1)$

$\underline{v}_e = \underline{n}_S \times \underline{n}_T = \left| \begin{array}{ccccc} i & j & k & i & j \\ 2 & 3 & 4 & 2 & 3 \\ -1 & 4 & 1 & -1 & 4 \end{array} \right| =$

$(3-16, -4-2, 8-(-3)) = (-13,-6,11)$

e egy pontja: Legyen $z=0$ S és T egyenletében (azaz azt keressük, hogy a $z=0$ síknak és e-nek hol van a metszete):

$$\left.\begin{array}{l} 2x+3y=5 \\ -x+4y=7 \;/\cdot 2 \\ -2x+8y=14 \end{array}\right\} (+)\; 11y=19$$

$y = \frac{19}{11}$

$x = 7-4y = \frac{1}{11}$

e egy pontja: $P\left(\frac{1}{11}, \frac{19}{11}, 0\right)$

e paraméteres egyenletrendszere:

$x = \frac{1}{11} - 13t$

$y = \frac{19}{11} - 6t \qquad t \in \mathbb{R}$

$z = 0 + 11t$

2. megoldás: Egy másik pontját az e egyenesnek úgy kapjuk meg, hogy $z=1$-et választunk: $2x+3y+4=5$

$-x+4y+1=7$

$$\left.\begin{aligned} 2x+3y&=1 \\ -x+4y&=6 \quad /\cdot 2 \\ -2x+8y&=12 \end{aligned}\right\} \oplus \quad 11y = 13$$

$$y=\frac{13}{11} \Rightarrow x = 6-4y = \frac{14}{11}$$

$Q\left(\frac{14}{11}, \frac{13}{11}, 1\right)$

$\underline{v} = \overrightarrow{PQ} = \left(\frac{13}{11}, \frac{6}{11}, 1\right)$

$\uparrow$ az 1. megoldásból

Az e egyenes paraméteres egyenletrendszere: (P egy pontja):

$$x = \frac{1}{11} + \frac{13}{11}s$$

$$y = \frac{19}{11} + \frac{6}{11}s \qquad s \in \mathbb{R}$$

$$z = 0 + 1\cdot s$$

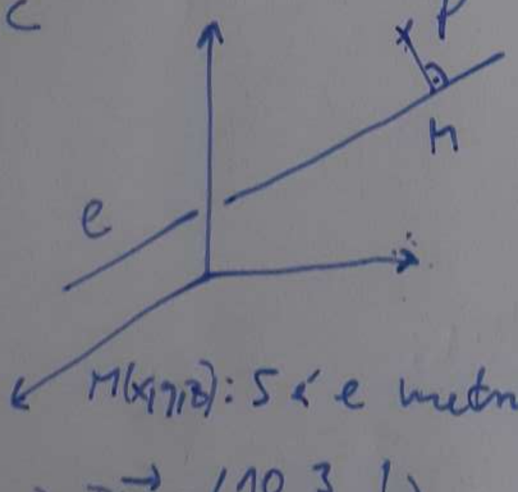
7/b $\quad \underline{v} = (-1,3,1)$

$$x = 3-t$$

$$y = 2+3t \qquad t\in\mathbb{R}$$

$$z = 1+t$$

7/c



$d = |\overrightarrow{PM}|$

Legyen S a P-t tartalmazó e-re merőleges sík: $\underline{n}_S = \underline{v}_e = (-1,3,1)$

$$-1(x-3)+3(y-2)+1(z-1)=0$$

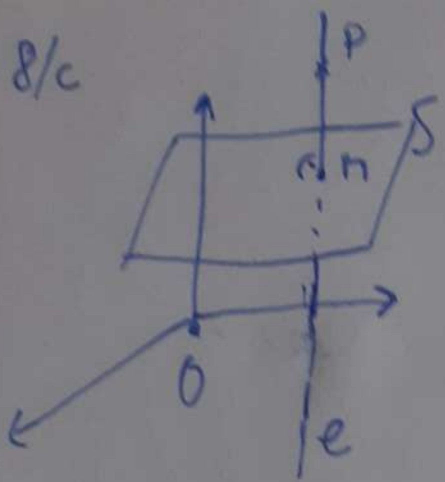
$$-x+3y+z=4$$

$M(x,y,z)$: S és e metszete: $-(4-t)+3(2+3t)+1+t=4$

$$11t = 1 \Rightarrow t = \frac{1}{11} \Rightarrow M\left(\frac{43}{11}, \frac{25}{11}, \frac{12}{11}\right)$$

$$\Rightarrow \overrightarrow{PM} = \left(\frac{10}{11}, \frac{3}{11}, \frac{1}{11}\right) \Rightarrow d = |\overrightarrow{PM}| = \sqrt{\left(\frac{10}{11}\right)^2+\left(\frac{3}{11}\right)^2+\left(\frac{1}{11}\right)^2} = \sqrt{\frac{110}{121}}$$

8/c



$\underline{v}_e = \underline{n}_S = (2,-1,3)$

$e:\ x = 2+2t$

$\quad y = -1-t$

$\quad z = 3+3t$

$M = S \cap e:\ 2(2+2t) - (-1-t) + 3(3+3t) = 5$

$14t = -9$

$t = -\frac{9}{14}$

$M = \left(2+2\left(-\frac{9}{14}\right), -1-\left(-\frac{9}{14}\right), 3+3\left(-\frac{9}{14}\right)\right) = \left(\frac{10}{14}, -\frac{5}{14}, \frac{15}{14}\right)$

$d = \overrightarrow{PM} = \sqrt{\left(-\frac{18}{14}\right)^2 + \left(\frac{9}{14}\right)^2 + \left(-\frac{27}{14}\right)^2} = \sqrt{\frac{1134}{196}} = \sqrt{\frac{81}{14}}$

$\overrightarrow{PM} = \left(-\frac{18}{14}, \frac{9}{14}, -\frac{27}{14}\right)$