Convergence of ergodic averages for many group rotations

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Suppose that G is a compact Abelian topological group, m is the Haar measure on G and $f : G \to \mathbb{R}$ is a measurable function. Given (n_k), a strictly monotone increasing sequence of integers we consider the nonconventional ergodic/Birkhoff averages

$M_N^\alpha f(x) = \frac{1}{N+1} \sum_{k=0}^{N} f(x + n_k\alpha).$

The f-rotation set is

$\Gamma_f = \{\alpha \in G : M_N^\alpha f(x) \text{ converges for } m \text{ a.e. } x \text{ as } N \to \infty.\}$

We prove that if G is a compact locally connected Abelian group and $f : G \to \mathbb{R}$ is a measurable function then from $m(\Gamma_f) > 0$ it follows that $f \in L^1(G)$.

A similar result is established for ordinary Birkhoff averages if $G = Z_p$, the group of p-adic integers.

However, if the dual group, $\widehat{G}$ contains "infinitely many multiple torsion" then such results do not hold if one considers non-conventional Birkhoff averages along ergodic sequences.

What really matters in our results is the boundedness of the tail, $f(x + n_k\alpha)/k$, $k = 1, ...$ for a.e. x for many α, hence some of our theorems are stated by using instead of Γ_f slightly larger sets, denoted by $\Gamma_{f,b}$. This is a joint work with G. Keszthelyi.