Michał Rams: Inhomogeneous Diophantine approximations

Abstract: We consider the shrinking target problem for irrational rotations, for a natural class of targets being balls centered at zero (with monotonically decreasing radii). That is, given an irrational number θ and a nonincreasing positive sequence $\varphi(n)$ we investigate the set $E_\varphi(\theta) = \{y \in \mathbb{R}, ||y + n\theta|| < \varphi(n) \text{ i.o. }\}$, where $||\cdot||$ denotes the distance from nearest integer. Our goal is calculating the Hausdorff dimension of $E_\varphi(\theta)$.

This question was first asked by Bugeaud and by Schmeling, Troubetzkoy, they (independently) solved it for $\varphi(n) = n^{-\gamma}$. I will present the full solution of the problem. What's interesting, while in the Bugeaud-Schmeling-Troubetzkoy case the answer does not depend on θ, in the general situation it is not the case: the Diophantine properties of θ play a critical role.

This is a joint work with Dong Han Kim and Baowei Wang.