BSCA1 2-24 2011 nov A csoport és B csoport

1A. $x(t_0) = 4 \cdot \mathrm{ch}\ln 2 = 4\,\frac{e^{\ln 2}+e^{-\ln 2}}{2} = 4\cdot\frac{2+\frac{1}{2}}{2} = 4\cdot\frac{4+1}{4} = 4\cdot\frac{5}{4} = 5$ (1)

$y(t_0) = 8\cdot \mathrm{sh}\ln 2 = 8\,\frac{e^{\ln 2}-e^{-\ln 2}}{2} = 8\cdot\frac{2-\frac{1}{2}}{2} = 8\cdot\frac{4-1}{4} = 8\cdot\frac{3}{4} = 6$ (1)

$\dot{x}(t) = 4\,\mathrm{sh}\,t \qquad \dot{x}(t_0) = 4\cdot\mathrm{sh}\ln 2 = 4\cdot\frac{3}{4} = 3$ (1) $\qquad \frac{\dot{y}(t_0)}{\dot{x}(t_0)} = \frac{10}{3}$

$\dot{y}(t) = 8\,\mathrm{ch}\,t \qquad \dot{y}(t_0) = 8\cdot\mathrm{ch}\ln 2 = 8\cdot\frac{5}{4} = 10$ (1)

$y - y(t_0) = \frac{\dot{y}(t_0)}{\dot{x}(t_0)}(x - x(t_0)) \Rightarrow y - 6 = \frac{10}{3}(x-5)$ (1)

2A. $f'(x) = \left(4e^{\frac{1}{x}} + (4x+8)\,e^{\frac{1}{x}}\left(-\frac{1}{x^2}\right)\right) = -\frac{e^{\frac{1}{x}}}{x^2}\,4(x^2 - x - 2) = 0$ (1) $\Leftrightarrow x_{12} = \frac{+1 \pm \sqrt{1+8}}{2}$ ($\sqrt{1+8} = 3$) $\nearrow -1$, $\searrow 2$

	$(-\infty,-1)$	-1	$(-1,0)$	$(0,2)$	2	$(2,\infty)$
f'	−	0	+	+	0	−
f	↘	l.min	↗	↗	lmax	↘

(2) (2)

3A. $\int 3x e^{-4x}\,dx = 3\int x e^{-4x}\,dx = 3\left\{-\frac{x}{4}e^{-4x} - \int\frac{e^{-4x}}{-4}\,dx\right\}$ (2) $= 3\left\{-\frac{x}{4}e^{-4x} - \frac{e^{-4x}}{16}\right\} + C$ (2)

$x = u \qquad e^{-4x} = v' \qquad v = \frac{e^{-4x}}{-4}$ (1)

(*) Ha a parc. integrálást elrontja az 1. lépésben, akkor max 2 pont adható

4A. $\frac{1}{x(x+5)} = \frac{A}{x} + \frac{B}{x+5} = \frac{Ax+5A+Bx}{x(x+5)} \Rightarrow \begin{array}{l} A+B=0 \\ 5A = 1 \Rightarrow A = \frac{1}{5} \qquad B = -\frac{1}{5}\end{array}$

$\int\frac{1}{x(x+5)}\,dx = \frac{1}{5}\int \frac{1}{x} - \frac{1}{x+5}\,dx = \frac{1}{5}\left\{\ln x - \ln(x+5)\right\} + C$

1B. $x(t_0) = 16\,\mathrm{sh}\ln 4 = 16\,\frac{e^{\ln 4}-e^{-\ln 4}}{2} = 16\cdot\frac{4-\frac{1}{4}}{2} = 16\,\frac{16-1}{8} = 16\cdot\frac{15}{8} = 30$ (1)

$y(t_0) = 8\,\mathrm{ch}\ln 4 = 8\cdot\frac{e^{\ln 4}+e^{-\ln 4}}{2} = 8\cdot\frac{4+\frac{1}{4}}{2} = 8\cdot\frac{16+1}{8} = 8\cdot\frac{17}{8} = 17$ (1)

$\dot{x}(t) = 16\,\mathrm{ch}\,t \qquad \dot{x}(t_0) = 16\cdot\mathrm{ch}\ln 4 = 16\cdot\frac{17}{8} = 34$ (1) $\qquad \frac{\dot{y}(t_0)}{\dot{x}(t_0)} = \frac{15}{34}$

$\dot{y}(t) = 8\,\mathrm{sh}\,t \qquad \dot{y}(t_0) = 8\cdot\mathrm{sh}\ln 4 = 8\cdot\frac{15}{8} = 15$ (1)

$y - y(t_0) = \frac{\dot{y}(t_0)}{\dot{x}(t_0)}(x - x(t_0)) \Rightarrow y - 17 = \frac{15}{34}(x-30)$ (1)

2B $f'(x) = 2e^{-\frac{1}{x}} + (2x-12)e^{-\frac{1}{x}}\left(\frac{1}{x^2}\right) = 2\,\frac{e^{-\frac{1}{x}}}{x^2}(x^2+x-6) = 0$ (1) $\Leftrightarrow x_{12} = \frac{-1 \pm\sqrt{1+24}}{2}$ ($\sqrt{1+24} = 5$) $\nearrow -3$, $\searrow +2$

	$(-\infty,-3)$	-3	$(-3,0)$	$(0,2)$	2	$(2,\infty)$
f'	+	0	−	−	0	+
f	↗	lmax	↘	↘	l.min	↗

(2) (2)

3B $\int -2x e^{8x}\,dx = -2\int x e^{8x}\,dx = -2\left\{\frac{x}{8}e^{8x} - \int\frac{e^{8x}}{8}\,dx\right\}$ (2) $= -2\left\{\frac{x}{8}e^{8x} - \frac{e^{8x}}{64}\right\} + C$ (2)

$x = u \qquad e^{8x} = v' \qquad v = \frac{e^{8x}}{8}$ (1) és (*) alkalmaz!

4B $\frac{1}{x(x-7)} = \frac{A}{x} + \frac{B}{x-7}$ (1) $= \frac{Ax-7A+Bx}{x(x-7)} \qquad \begin{array}{l} A+B=0 \\ -7A = 1 \Rightarrow A = -\frac{1}{7} \qquad B = \frac{1}{7}\end{array}$

$\int\frac{1}{x(x-7)}\,dx = -\frac{1}{7}\int \frac{1}{x} - \frac{1}{x-7}\,dx$ (2) $= -\frac{1}{7}\left\{\ln x - \ln(x-7)\right\} + C$ (2)