BSCA1 2. ZH pótló 2011. december

1. $x(t) = \sin^2 t \cos t \qquad x(-\frac{\pi}{4}) = (\frac{\sqrt{2}}{2})^2 \cdot \frac{\sqrt{2}}{2} = \frac{1}{4}\cdot\sqrt{2}$ } (1)

$y(t) = \sin^2 t \sin t \qquad y(-\frac{\pi}{4}) = (\frac{\sqrt{2}}{2})^2 (-\frac{\sqrt{2}}{2}) = -\frac{1}{4}\sqrt{2}$ }

$\dot{x}(t) = 2\sin t\cos^2 t - \sin^3 t$ (1)

$\dot{y}(t) = 2\sin^2 t\cos t + \sin^2 t\cos t$ (1)

$\frac{\dot{y}}{\dot{x}} = \frac{2\sin^2 t\cos t + \sin^2 t\cos t}{2\sin t\cos^2 t - \sin^3 t} = \frac{3\sin^2 t\cos t}{\sin t(2\cos^2 t - \sin^2 t)}$ (1)

$\frac{\dot{y}}{\dot{x}}\Big|_{-\frac{\pi}{4}} = \frac{3\sin t\cos t}{2\cos^2 t - \sin^2 t}\Big|_{-\frac{\pi}{4}} = \frac{\frac{3}{2}\sin 2t}{2\cos^2 t - \sin^2 t}\Big|_{-\frac{\pi}{4}} = \frac{-1\cdot\frac{3}{2}}{2\cdot\frac{1}{2} - \frac{1}{2}} = \frac{-1\cdot 3}{\frac{1}{2}\cdot 2} = -3$

$y - y_0 = f'(x_0)(x - x_0) \Rightarrow y + \frac{\sqrt{2}}{4} = -3(x - \frac{\sqrt{2}}{4})$ (1) $\Rightarrow y = -3x + \frac{\sqrt{2}}{2}$

2. $f'(x) = \frac{1}{(1+x^2)}\cdot 2x$ (1)

$f''(x) = 2\,\frac{(1+x^2) - x\cdot 2x}{(1+x^2)^2} \overset{(1)}{=} \frac{2}{(1+x^2)^2}(1-x^2) = 0 \Leftrightarrow x_{1,2} = \pm 1$ (1)

	$(-\infty,-1)$	-1	$(-1,1)$	1	$(1,\infty)$
f''	$-$	0	$+$		$-$
f	$\frown$	infl	$\smile$	infl	$\frown$

(1) (1)

3. $\int x \,\mathrm{arctg}\, x^2\, dx = \frac{x^2}{2}\mathrm{arctg}\, x^2 - \int \frac{x^2}{2}\frac{1}{1+x^4}\cdot 2x \overset{(2)}{=} \frac{x^2}{2}\mathrm{arctg}\, x^2 - \frac{1}{4}\ln(1+x^4) + C$ (2)

$u' = x \quad u = \frac{x^2}{2} \quad v = \mathrm{arctg}\, x^2 \quad v' = \frac{1}{1+x^4}\cdot 2x$ (1)

Ha elrontja az 1. lépésben a parc. integrálást akkor max 2 pont adható.

4. $\int \frac{1}{x^3+x}\, dx = \int \frac{1}{x} - \frac{x}{x^2+1}\, dx \overset{(2)}{=} \ln x - \frac{1}{2}\ln(x^2+1) + C$ (2)

$\frac{1}{x^3+x} = \frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ (1) $= \frac{Ax^2+A+Bx^2+Cx}{x(x^2+1)}$

$A+B = 0 \Rightarrow B = -1$

$A = 1$

$C = 0$