Conference on
"Hyperbolic Dynamics and Statistical Physics"
dedicated to the 75th birthday of Domokos Szász
May 17 – May 21, 2016

organized by
Péter Bálint (TU Budapest), Péter Nándori (U Maryland), Bálint Tóth (U Bristol and TU Budapest), Imre Péter Tóth (TU Budapest)

within the framework of the
ESI Programme on
"Mixing Flows and Averaging Methods"
April 4 – May 25, 2016

organized by
Peter Bálint (TU, Budapest), Henk Bruin (U Vienna), Carlangelo Liverani (U Rome, Tor Vergata), Ian Melbourne (U Warwick), Dalia Terhesiu (U Exeter)

- **Tuesday, May 17, 2016**

  8:30 – 9:00 **Registration**

  9:00 – 9:10 **Opening**

  9:10 – 10:00 **Klaus Schmidt**

  10:00 – 10:30 *coffee / tea break*

  10:30 – 11:20 **Konstatin Khanin**

  11:30 – 12:20 **Jens Marklof**

  12:20 – 14:30 *lunch break*

  14:30 – 15:20 **Dmitry Dolgopyat**

  15:20 – 15:50 *break*

  15:50 – 16:40 **Nándor Simányi**

- **Wednesday, May 18, 2016**

  9:00 – 9:50 **Benjamin Weiss**

  9:50 – 10:20 *coffee / tea break*

  10:20 – 11:10 **Thomas Gilbert**

  11:20 – 12:10 **Carl Dettmann**

  12:20 – 14:30 *lunch break*

  14:30 – 15:20 **Bálint Tóth**

  15:20 – 15:50 *break*

  15:50 – 16:40 **Stefano Olla**

- **Thursday, May 19, 2016**

  9:00 – 9:50 **Mark Demers**

  9:50 – 10:20 *coffee / tea break*

  10:20 – 11:10 **Leonid Bunimovich**

  11:20 – 12:10 **Ian Melbourne**

  12:20 – 14:30 *lunch break*

  14:30 – 15:20 **Károly Simon**

  15:20 – 15:50 *break*

  15:50 – 16:40 **Carlangelo Liverani**

- **Friday, May 20, 2016**

  9:00 – 9:50 **Viviane Baladi**

  9:50 – 10:20 *coffee / tea break*

  10:20 – 11:10 **Péter Nándori**

  11:20 – 12:10 **Zsolt Pajor-Gyulai**

  12:20 – 14:30 *lunch break*

  14:30 – 15:20 **László Erdős**

  15:20 – 15:50 *break*

  15:50 – 16:40 **Yuri Kifer**

  19:00 – **Conference dinner**

- **Saturday, May 21, 2016**

  ! The program on May 21 takes place at the Faculty of Mathematics, Sky Lounge !

  9:00 – 9:50 **Henk Bruin**

  9:50 – 10:20 *coffee / tea break*

  10:20 – 11:10 **Maciej Wojtkowski**

  11:20 – 12:10 **Lai-Sang Young**

  12:20 – 14:30 *lunch break*

  14:30 – 15:20 **Yakov Pesin**

  15:20 – 15:50 *break*

  15:50 – 16:40 **Anatole Katok**

The talks on May 17-20 take place at the ESI, Boltzmann Lecture Hall

Titles and abstracts

- **Viviane Baladi**: "Exponential decay of correlations for Sinai billiard flows" (joint work with M. Demers and C. Liverani.)
  Abstract: In 1998, Young obtained exponential decay of correlations for the finite-horizon discrete-time Sinai billiard map on the 2-torus. The continuous-time billiard flow is much more difficult to handle: Melbourne had obtained superpolynomial upper bounds (2007) and Chernov stretched exponential upper bounds (2007). In 2011, Demers and Zhang published a new functional proof (spectral gap without Markov partition nor tower) of Youngs exponential decay result for discrete-time Sinai billiards. Combining this work of Demers-Zhang with the techniques introduced by Baladi-Liverani (2012) to study a piecewise hyperbolic contact flow model, we prove exponential decay for the two-dimensional finite horizon Sinai billiard flow. The key tool to obtain the spectral gap is the construction of approximate stable/unstable foliations (in the kernel of the contact form) at a fixed scale.

- **Henk Bruin**: "The Dolgopyat inequality for BV observables"
  Abstract: In this joint work with Dalia Terhesiu, we give estimates on the norm of twisted transfer operators, known as the "Dolgopyat inequality". The underlying system is a non-Markov AFU map on the interval, and the Banach space of observables is BV.

  Whilst following largely the approach of Baladi & Vallée (extended by Avila et al. and Araújo & Melbourne), the focus will be on what needs to be implemented to deal with the discontinuties that non-Markov maps generate.

- **Leonid Bunimovich**: "Isospectral transformations of multidimensional systems and networks"
  Abstract: I will discuss a new approach to analysis of networks. It proved to be efficient for various applications. Some examples will be demonstrated.

- **Mark Demers**: "Escape rates and limiting distributions for intermittent maps with holes"
  Abstract: Dynamical systems with holes model systems in which mass or energy is allowed to escape over time and have attracted much attention over the last ten years. Typically, one starts with a closed system and declares a subset of the phase space to be the 'hole,' essentially an absorbing set. To date, most published works focus on systems in which the rate of mixing, and thus the rate of escape, are exponential. This talk will investigate a class of polynomially mixing systems with holes which exhibit qualitatively different behavior from exponentially mixing systems; this behavior can be characterized as a loss of stability from the point of view of the absolutely continuous invariant measure for the closed system. By varying the potential of the associated transfer operator in this family, we are able to classify three distinct regimes with different limiting behaviors based on a relation between the rate of mixing and the rate of escape. Finally, we attempt to recover some notion of stability via the measures supported on the survivor set of the open system, even when no limiting absolutely continuous conditionally invariant measure exists. This is joint work with Bastien Fernandez, CNRS, and Mike Todd, St. Andrews.

- **Carl Dettmann**: "How sticky is the chaos/order boundary?"
  Abstract: In dynamical systems with divided phase space, the vicinity of the boundary between regular and chaotic regions is often "sticky," that is, trapping orbits from the chaotic region for long times. Here, we investigate the stickiness in the simplest mushroom billiard, which has a smooth such boundary, but surprisingly subtle behaviour. As a measure of stickiness, we investigate $P(t)$, the probability of remaining in the mushroom cap for at least time t given uniform initial conditions in the chaotic part of the cap. The stickiness is sensitively dependent on the radius of the stem r via the Diophantine properties of $\rho = (2/\pi)\, arccos\, r$. Almost all ρ give rise to families of marginally unstable periodic orbits (MUPOs) where $P(t) \sim C/t$, dominating the stickiness of the boundary. After characterising the set for which rho is MUPO-free, we consider the stickiness in this case, and where rho also has continued fraction expansion with bounded partial quotients. We show that $t^2 P(t)$ is bounded, varying infinitely often between

values whose ratio is at least 32/27. When ρ has an eventually periodic continued fraction expansion, that is, a quadratic irrational, $t^2P(t)$ converges to a log-periodic function. In general, we expect less regular behaviour, with upper and lower exponents lying between 1 and 2. The results may shed light on the parameter dependence of boundary stickiness in annular billiards and generic area preserving maps.

- **Dmitry Dolgopyat**: "Piecewise linear Fermi-Ulam pingpongs"
  Abstract: We consider a particle moving freely between two periodically moving infinitely heavy walls. We assume that one wall is fixed and the second one moves with piecewise linear velocities. We study the question about existence and abundance of accelerating orbits for that model. This is a joint work with Jacopo de Simoi.

- **László Erdős**: "Local law of addition of random matrices"
  Abstract: Let A and B be two N by N deterministic real diagonal matrices and U be an N by N Haar distributed unitary matrix. It is well known since Voiculescu's seminal work that the spectral distribution of the sum of random matrices $H = A + UBU^*$ converges weakly to the free additive convolution of the spectral distributions of A and B, when N tends to infinity. In this talk we establish the optimal rate $\frac{1}{N}$ for this convergence. This is a joint work with Zhigang Bao and Kevin Schnelli.

- **Thomas Gilbert**: "Revisiting the derivation of Fourier's law of heat conduction in the Kipnis-Marchioro-Presutti model"
  Abstract: I will revisit the exactly solvable KMP model of heat flow [J. Stat. Phys. 27 65 (1982)] and describe a systematic characterisation of its non-equilibrium stationary states for systems of arbitrary sizes. In particular, the arguments yield a straightforward derivation of Fourier's law, different from that presented in the original KMP study.

- **Anatole Katok**: "Flexibility program for quantitative properties of hyperbolic dynamical systems"
  Abstract: Most efforts in the study of systems with hyperbolic/stochastic/chaotic properties go into establishing qualitative properties or, in the case of numerical characteristics, such as entropies or Lyapunov characteristic exponents, into estimating their values. There are certain general relations among dynamical properties of classical dynamical systems such as the variational principle for entropy, or Pesin entropy formula as well as some relations between dynamics and topology, such as inequalities between topological entropy and growth in the fundamental group for continuous systems and growth in homology for smooth systems. Precise calculation are possible only in very few cases, primarily of algebraic nature such as homogeneous or affine systems. The general paradigm of flexibility can be rather vaguely formulated as follows:

  Under properly understood general restrictions within a fixed class of smooth dynamical systems quantitative dynamical invariants take arbitrary values.

  Most known constructions are perturbative and hence at best would allow to cover a small neighborhood of the values allowed by the model, or more often, not even that, since homogeneous systems are often "extremal". So establishing flexibility calls for *non-perturbative or large perturbation constructions* in large families to cover possible values of invariants.

  Work on flexibility is still in its infancy and in many situations the proper "general restrictions" are not fully understood. In this talk I will discuss general conjectures and first two results that confirm those conjectures (technically still in progress since preprints are not publicly available): one (joint with J. Bochi and F. Rodriguez Hertz) deals with description of possible values of Lyapunov exponents for Anosov volume preserving diffeomorphisms of a torus and the other (joint with A.Erchenko) describes all possible pairs of values for Liouville and topological entropy for geodesic flows on compact surfaces of negative curvature.

- **Konstantin Khanin**: "On KPZ universality and renormalization."
  Abstract: We shall discuss a new geometrical renormalization scheme related to the problem of universality of the KPZ (Kardar-Parisi-Zhang) phenomenon.

- **Yuri Kifer**: "Some extensions of the Erdős–Rényi Law of Large Numbers"
  Abstract: I will talk about extensions of the Erdős-Rényi law of large numbers, in particular, to the averaging setup.

- **Carlangelo Liverani**: "Random Lorentz Gas"
  Abstract: Contrary to the periodic Lorentz gas, in the random case very few rigorous results are known. I will present some debatable personal speculations and overoptimistic ideas toward studying such a model. I'll pretend to substantiate them by discussing a toy model studied in collaboration with Romain Aimino.

- **Jens Marklof**:"Spherical averages in the space of marked lattices"
  Abstract: A marked lattice is a d-dimensional Euclidean lattice, where each lattice point is assigned a mark via a given random field on d. We prove that, if the field is strongly mixing with a faster-than-logarithmic rate, then for every given lattice and almost every marking, large spheres become equidistributed in the space of marked lattices. A key aspect of our study is that the space of marked lattices is not a homogeneous space, but rather a non-trivial fiber bundle over such a space. As an application, we prove that the free path length in a periodic Lorentz gas with random defects has a limiting distribution in the Boltzmann-Grad limit. This is joint work with Ilya Vinogradov (Princeton).

- **Ian Melbourne**: "Mixing for infinite measure flows"
  Abstract: In joint work with Dalia Terhesiu, we introduced an operator renewal equation for continuous time dynamical systems. This equation has strong similarities to, and is almost as tractable as, the discrete operator renewal equation of Gouezel and Sarig. Using this equation, we show how results on mixing for infinite measure maps carry over to infinite measure flows.

- **Péter Nándori**:"The first encounter of two billiard particles of small radius"
  Abstract: We prove that the time of the first collision between two particles on a Sinai billiard table converges weakly to an exponential distribution when time is rescaled by the inverse of the radius of the particles. This results provides a first step in studying the energy evolution of hard ball systems in the rare interaction limit. Joint work with Dmitry Dolgopyat.

- **Stefano Olla**:"Energy transport in acoustic and non-acoustic chains."
  Abstract: One-dimensional chains of oscillators are one of the most simple models where to study macroscopic transport of energy and convergence to equilibrium. The sound speed of the chain is given by the derivative of the thermodynamic equilibrium tension as a function of the average length of the chain. If this is not null, the chain is said acoustic, otherwise non-acoustic. In acoustic chains we expect a separation of time scale between the approach to mechanical equilibrium (uniform tension) and thermal equilibrium (uniform temperature). Mechanical equilibrium is achieved in a hyperbolic (ballistic) time scale governed by Euler equations, while thermal equilibrium is achieved in a longer superdiffusive time scale, governed by a fractional heat equation. This superdiffusion of energy is due to a divergence of the thermal conductivity. This picture can be proven mathematically for harmonic acoustic chain with random collisions, and there is a wide numerical evidence for non-linear chains. In the corresponding non-acoustic harmonic chain with random collisions there is not such separation of scales. The macroscopic limits of the energy density, momentum and the curvature (or bending) of the chain satisfy, in a diffusive space-time scaling, an autonomous system of equations. The curvature and momentum evolve following a linear system that corresponds to a damped Euler-Bernoulli beam equation. The macroscopic energy density evolves following a non linear diffusive equation. In particular the energy transfer is diffusive in this dynamics. This provides a first rigorous example of a normal diffusion of energy in a one-dimensional dynamics that conserves the momentum.

- **Yakov Pesin**:"Building thermodynamics for non-uniformly hyperbolic maps"
  Abstract: The talk is a brief survey of the current state of the theory of thermodynamic formalism for non-uniformly hyperbolic systems. I will describe several recently developed approaches. The

first of these involves Markov models such as Young towers, countable-state Markov shifts, and inducing schemes. The other two are less fully developed but have seen significant progress in the last few years: these involve coarse-graining techniques (expansivity and specification) and geometric arguments involving push forward of densities on admissible manifolds.

- **Zsolt Pajor-Gyulai**:"Stochastic approach to anomalous diffusion in two dimensional, incompressible, periodic, cellular flows."
  Abstract: It is a well known fact that velocity grandients in a flow change the dispersion of a passive tracer. One clear manifestation of this phenomenon is that in systems with homogenization type diffusive long time/large scale behavior, the effective diffusivity often differs greatly from the molecular one. An important aspect of these well known result is that they are only valid on timescales much longer than the inverse molecular diffusivity. We are interested in what happens on shorter timescales (subhomogenization regimes) in a family of two-dimensional incompressible periodic flows that consists only of pockets of recirculations essentially acting as traps and infinite flowlines separating these where significant transport is possible. Our approach is to follow the random motion of a tracer particle and show that under certain scaling it resembles a time-changed Brownian motions. This shows that while the trajectories are still diffusive, the variance grows differently than linear.

- **Klaus Schmidt**: "Expansiveness, homoclinic and periodic points for algebraic Z^d-actions."
  Abstract: Every Laurent polynomial f in d variables with integer coefficients defines a Z^d-action α_f by automorphisms of a compact abelian group (in fact, of an infinite-dimensional torus or solenoid), and every Z^d-action by automorphisms of a compact abelian group with completely positive entropy can essentially be reduced to actions of this form.

  Many of the dynamical properties of such a Z^d-action α_f, like expansiveness, existence of homoclinic points, or the logarithmic growth rate of periodic points, are determined by the zeros (i.e., the complex variety $V(f)$) of the polynomial f. This connection is analogous to the correspondence between the eigenvalues of a matrix $A \in GL(n, Z)$ and the dynamical properties of the toral automorphism defined by A.

  This talk is based on joint work with Doug Lind and Evgeny Verbitskiy.

- **Nándor Simányi**: "Doma Szász and the Boltzmann-Sinai Hypothesis"
  Abstract: We survey the several fundamental ideas and contributions Doma made for the proof of the Boltzmann-Sinai Hypothesis.

- **Károly Simon**: "On the dimension of diagonally affine self-affine sets."
  (Joint work with Balázs Bárány and Michał Rams)
  Abstract: We consider self-affine IFS on the plane of the form $f_i(x) = A_i x + t_i, i = 1, ..., m$ where the matrices A_i are diagonal matrices of norm smaller than one. We combine methods of Mike Hochman with the Ledrappier-Young formula to compute the dimensions of the corresponding self-affine set and self-affine measures.

- **Bálint Tóth**: "Tagged particle diffusion in deterministic dynamics"
  Abstract: I will survey our joint work with Doma (done in the mid-eighties) on the diffusive behaviour of a Brownian particle suspended in an ideal gas. The problem is old and difficult. There are more questions than answers. It is worth looking at them from time to time.

- **Benjamin Weiss**: "Limit theorems for positive processes"
  Abstract: A very basic question is what are the possible limiting distributions of S_n/a_n where S_n is the sum of the first n random variables in a stationary ergodic process $\{X_i\}$ and $\{a_n\}$ is a sequence of normalizing constants. If $a_n = n$ then in earlier work with Jean-Paul Thouvenot we showed that any distribution on the real line is possible. However if one considers positive random variables then clearly the only possible limits are a constant - finite or infinite. For other sequences of constants a_n non-trivial limits are possible, e.g. positive stable distributions for an appropriate IID process. It turns out that here too ALL positive distributions are possible. There

are applications of this result to the ergodic theory of infinite measure preserving transformations. (joint work with Jon Aaronson)

- **Maciej P. Wojtkowski**: "The system of falling balls revisited"
  Abstract: The system of falling balls is a hamiltonian system of arbitrary number of particles in which hyperbolicity is present under some conditions on the masses. It was discovered some 25 years ago. Less known is the version of this system where no conditions on the masses are needed.

  We will discuss the remaining roadblocks to establishing the ergodicity of the last system.

- **Lai-Sang Young**: "Lyapunov exponents of some random dynamical systems"
  Abstract: As Lyapunov exponents measure the rates at which nearby orbits diverge, one might expect that geometric expansion or stretching in a map would lead to positive Lyapunov exponents. This, however, is very difficult to prove - except for maps with invariant cones. An example that has come to symbolize the challenge is the standard map: there is a parameter that corresponds to the amount of stretching, and for no value of this parameter, no matter how large, has anyone been able to prove the positivity of Lyapunov exponents. In this talk, I will explain the underlying issues, and present some results to illustrate what a tiny amount of noise can do.