.... Grade	..... Total points	Exam points	Midterm points

NAME: .... NEPTUN-CODE: ....

Exam — 2015-12-14
90 minutes - Both mathematical and Excel formulas are accepted

1.

(a) The probability of an event is 0.7. How many experiments are needed to guarantee that out of that many experiments the event will occur at least once with a probability greater than 0.9?

(b) Theoretical question. State at least 3 of the the multiplication rules true for 3 independent events. Denote the events by A, B, C.

2. The students in a class attend a lecture, independently of the others, with the same (unknown) probability. The expected value of the students present at the lecture is 25. What is the probability that there are more than 20 students at a lecture if the number of the students in the class

(a) is 40?

(b) is a large unknown number (say, more than hundred)?

Explain also why the distributions you use are justified.

3. Five people - independently of each other - arrive into a casino between midnight and 1 a.m. according to uniform distribution. Let X be the time instant when the second person arrives. Time is measured in hours, which means, for example: if the second arrives at $0:30$, then $X = 0.5$.

(a) Theoretical question. Derive the formula of the density function of X from the definition of X.

(b) Calculate the expected value of X.

4.

(a) X follows exponential distribution with an expected value 0.5. If $X = x$, then Y follows exponential distribution with parameter x. Determine the (unconditional) density function of Y.

(b) Theoretical question. State what the "Memoryless property" means, and show that the exponential distributions satisfy it.

5. Let us accept that the amount of time spent in work during a month and the monthly income of a randomly chosen worker in a large workshop follows a two-dimensional normal distribution. The expected value and the standard deviation of the amount of time are 170 hours and 10 hours. The same parameters for the income are 180 000 forints and 20 000 forints. Assume that the correlation coefficient is 0.6.

(a) What is the probability that a randomly chosen worker earns more than 185 000 Forints if he works 160 hours during a month?

(b) What is the probability that a randomly chosen worker works more than 160 hours during a month if he earns 185 000 Forints?

You may give your answers in terms of a proper Excel function or the distribution function ϕ of the standard normal distribution.

6.

(a) How would you simulate three independent events so that their probabilities are 0.6 and 0.7 and 0.8 respectively?

(b) Give a simulation formula to simulate the random variable X and the random variable Y of problem 4.