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Optimization Problems in Gas Transportation

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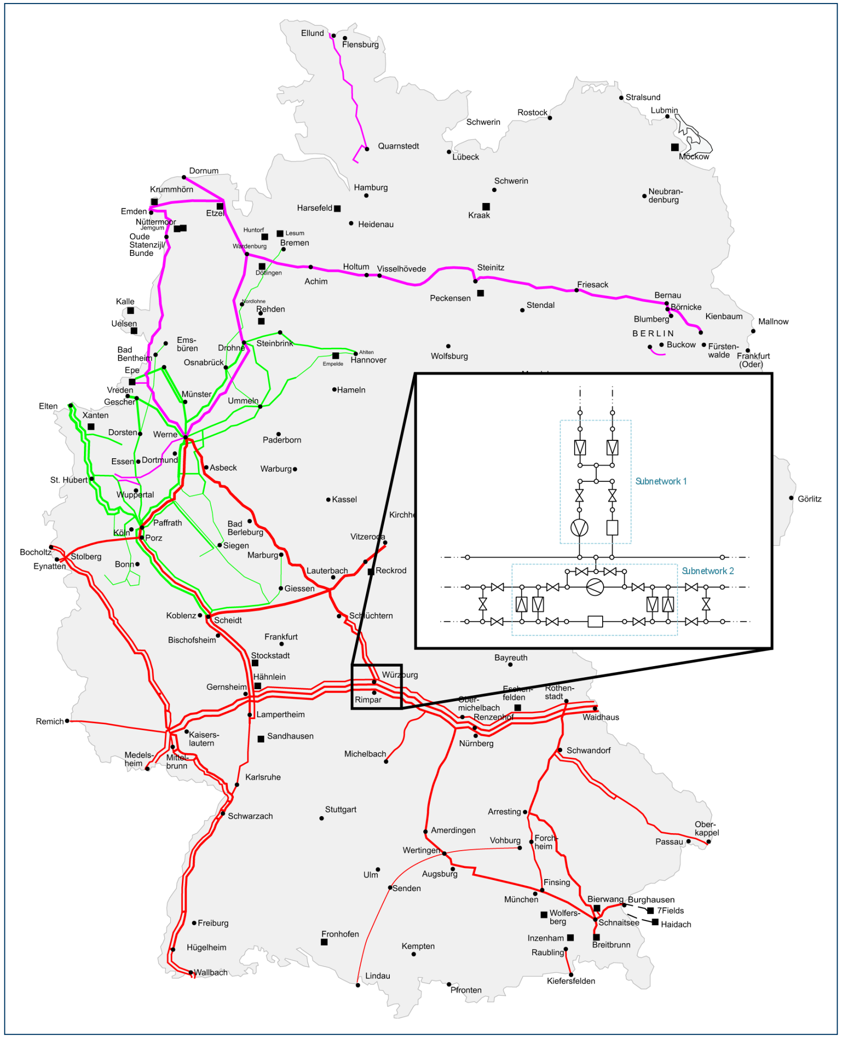
CWM^{3}EO 2014, Budapest

Why do we care?



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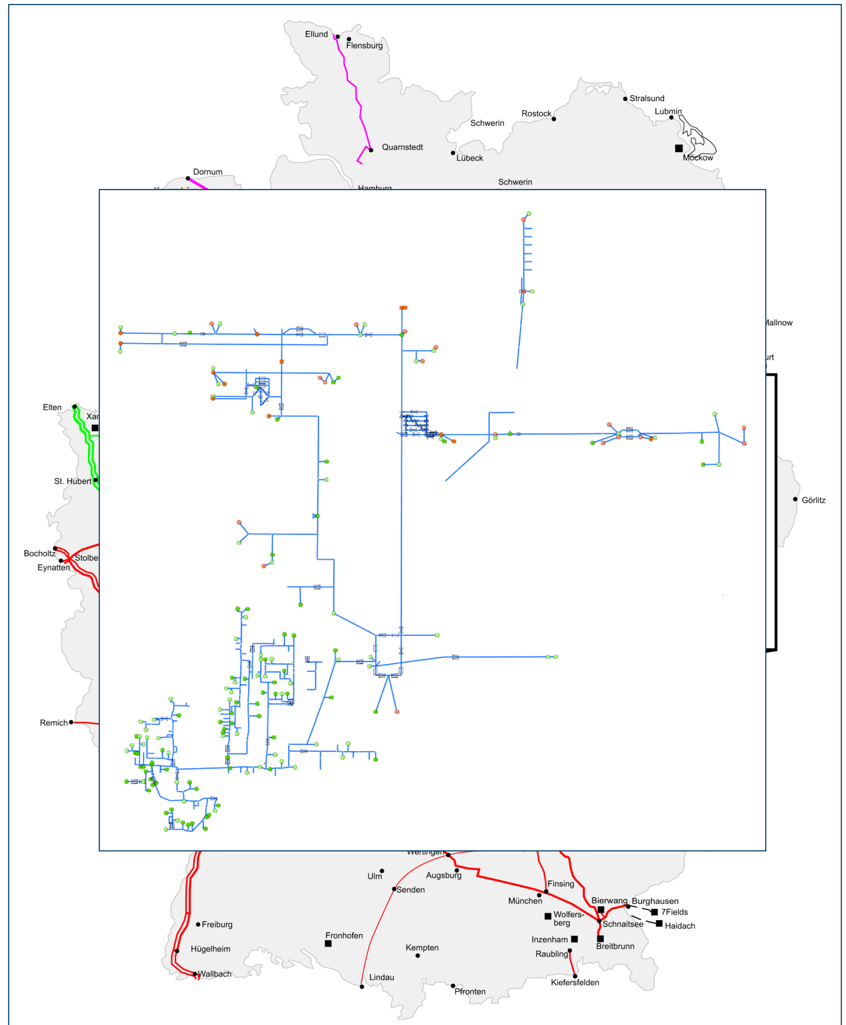

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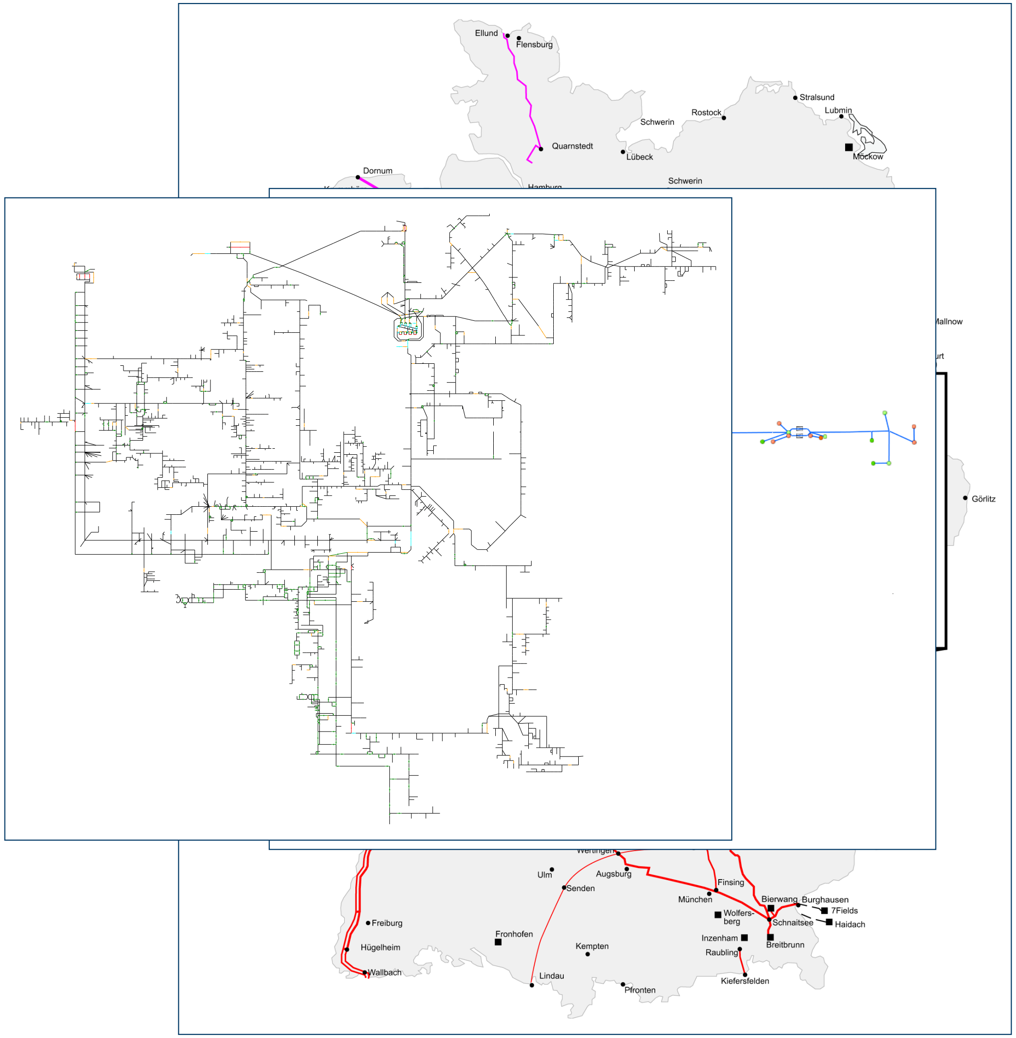

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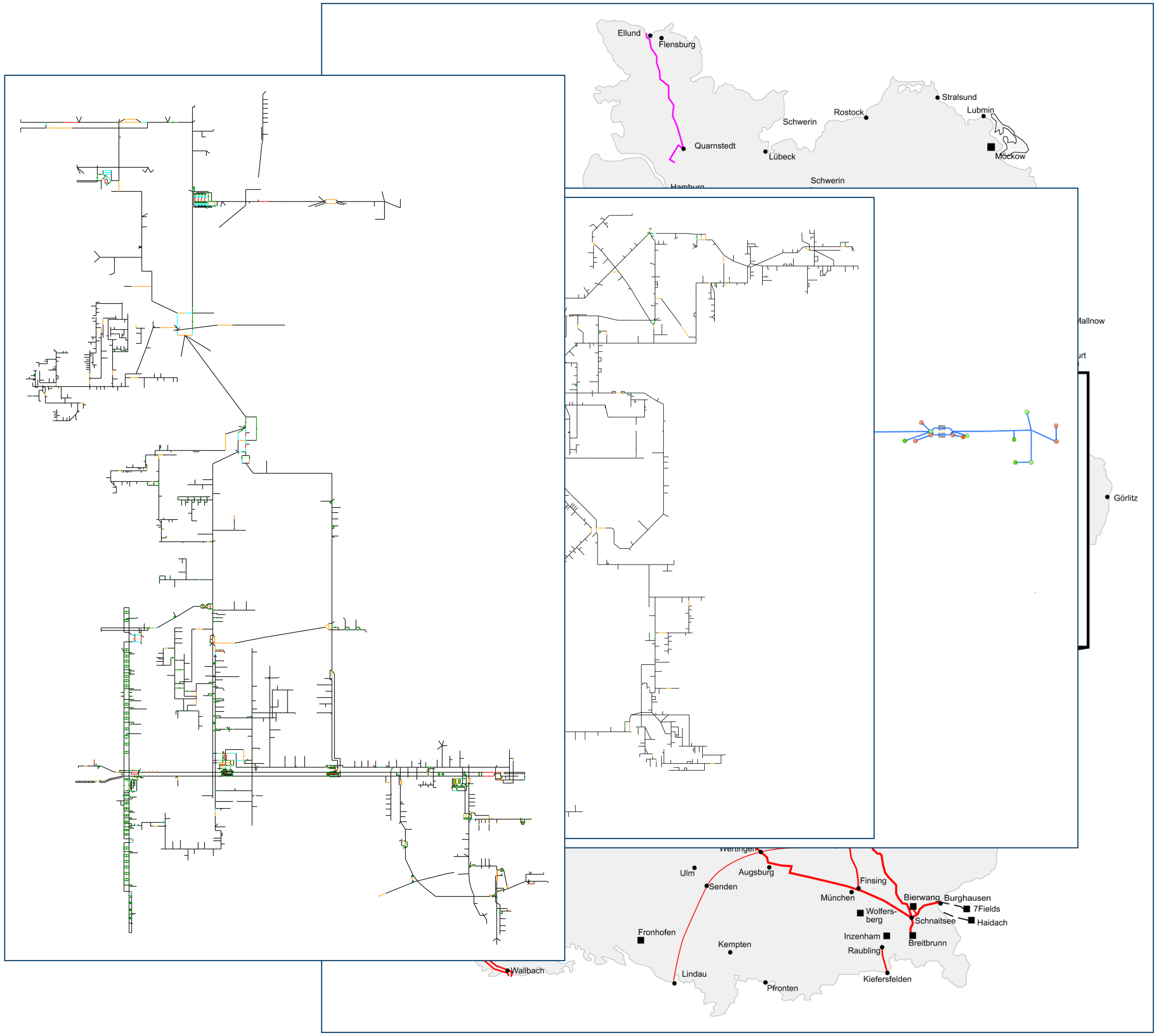

Why do we care?

Interesting MINLP

- Lots of structure,
- challenging nonlinearities,
- many problem variants.

Infeasibility important . . .

- needed to certify rejections

Good setting to test MINLP methods

Advertisement: Our book

Evaluating Gas Network Capacities

Koch, Hiller, Pfetsch, Schewe (Eds.),
to appear in the SIAM-MOS series, December 2014

Structure of the Project

Funding

- Open Grid Europe
- German Federal Government

Partners

- FAU Erlangen-Nuremberg
- Humboldt University Berlin
- Leibniz University Hannover
- TU Braunschweig
- University Duisburg-Essen
- Weierstraß Institute Berlin
- Zuse Institute Berlin

What are the problems?

Problems

Standard contract

"On any given day you are allowed to supply/demand up to X units of gas at node v if you have matching partners at some other nodes"

Problems

- Given a supply/demand situation on a given day, is it technically feasible?
- How large may X be?
- Given a set of nodes, how large may we choose each X_v such they can be satisfied simultaneously?
- If one of the above problems has no satisfying solution, where can we build a network extension to ameliorate the situation?

Validation of nominations – Prerequisites

Given

- (stationary) gas network
  complete specification of all pipes, compressors, valves, . . .
- active elements: compressors, valves, control valves, resistors
- temperature

Nomination

- Precise supply amounts at entry nodes
- Precise demands at exit nodes
- Pressure bounds at entries/exits
- Balance: flow in = flow out

Validation of nominations – Task

Given

- a specification of the network,
- a nomination, defining supplies/demands.

Task: Find

1. Settings for active elements
   (valves, control valves, compressors) and
2. values for physical network parameters,

which satisfy

- laws of physics (according to the model) and
- regulatory and technical requirements.

Booking validation

Given a new contract at node u, how large may we choose X_u, such that all supply/demand-scenarios can be satisfied?

- Is *not* a robust optimization problem,
- how to deal with the infinite scenario space?

Calculation of technical capacities in gas networks?

Given a set of nodes, how large may we choose each X_v, such that all supply/demand-scenarios can be satisfied?

Definition according to GasNZV

Technical capacity is the maximal fixed capacity which the network operator can offer guaranteeing system integrity and fulfiling all restrictions of network operation.

Problem: Capacity maximization

Goal

Tool to compute available technical capacity

Problem

Given:

- a technical specification of the gas network
- technical and contractual requirements

Find:

- *maximal* capacity,
- such that *all* (conforming) nominations can be transported.

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Validation of nominations: Physical modeling

Modeling – Discrete and Nonlinear

Nonlinear

- Pipes
- Compressors
- Resistors

Discrete

- Valves
- “Control Valves”
- Switching inside compressor stations

Network model

Basic model

Graph G with node set V and arc set A

Nodes

- Entries, exits, and inner nodes

Arcs

- Pipes, Resistors, ...

Quantities

- pressure p, mass flow q,
- density ρ, and temperature T

Hierarchical modeling

There is no true model

- need approximations,
- which typically come in hierarchies.

Are your solutions truely feasible?

- For which model?
- Does this translate to other models?
- Can you detect infeasibility?

Pipes

1d Euler equations for cylindrical pipes

$$\frac{\partial \rho}{\partial t} + \frac{1}{A}\frac{\partial q}{\partial x} = 0, \qquad \text{(continuity)}$$

$$\frac{1}{A}\frac{\partial q}{\partial t} + \frac{\partial p}{\partial x} + \frac{1}{A}\frac{\partial (q\,v)}{\partial x} + g\,\rho\,s + \lambda(q)\frac{|v|\ v}{2D}\rho = 0, \qquad \text{(momentum)}$$

$$A\,\rho\,c_p\left(\frac{\partial T}{\partial t} + v\frac{\partial T}{\partial x}\right) - A\left(1 + \frac{T}{z}\frac{\partial z}{\partial T}\right)\frac{\partial p}{\partial t} - A\,v\frac{T}{z}\frac{\partial z}{\partial T}\frac{\partial p}{\partial x} + A\,\rho\,v\,g\,s + \pi D\,c_{\mathrm{HT}}\,(T - T_{\mathrm{soil}}) = 0, \qquad \text{(energy)}$$

needed additionally

$$\rho\,R_s\,T\,z(p, T) = p. \qquad \text{(state)}$$

Plus . . .

Model for the real gas factor z

AGA-8 $z(p,T) = 1 + 0.257\,p_r - 0.533\,p_r/T_r,$

Papay $z(p,T) = 1 - 3.52p/p_c e^{-2.26T/T_c} + 0.247(p/p_c)^2 e^{-1.878T/T_c},$

or . . .

Model for the friction coefficient λ

Hagen-Poisseuille (laminar) $\lambda(q) = \frac{64}{\mathrm{Re}(q)},$

Prandtl-Colebrook (turb.) $\frac{1}{\sqrt{\lambda}} = -2\log_{10}\left(\frac{2.51}{\mathrm{Re}(q)\sqrt{\lambda}} + \frac{k}{3.71\,D}\right),$

or . . .

For our model: only stationary and isothermal . . .

After simplification: System can be solved!

$$p_v^2 = \left(p_u^2 - \Lambda \, |q| \, q \, \frac{e^S - 1}{S} \right) e^{-S}$$

Λ calculated from gas- and pipe-parameters
S calculated from the height difference of the nodes

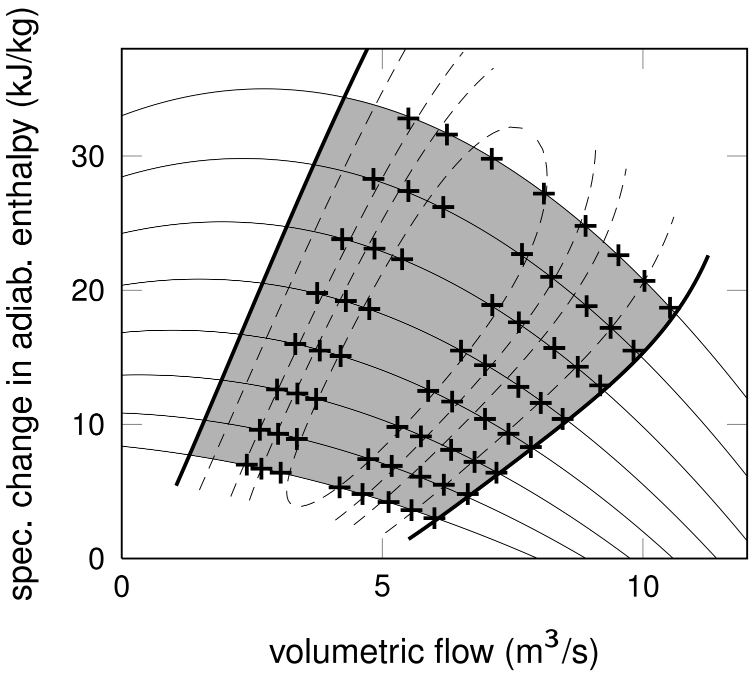
More precisely ...

$$\tilde{S} := \frac{2\, g\, s}{R_s\, z_m\, T_m}, \quad \tilde{\Lambda} := \lambda(q) \frac{R_s\, z_m\, T_m}{A^2\, D}.$$

Parameter uncertainties

Many of these parameters are only rough approximations

Compressors

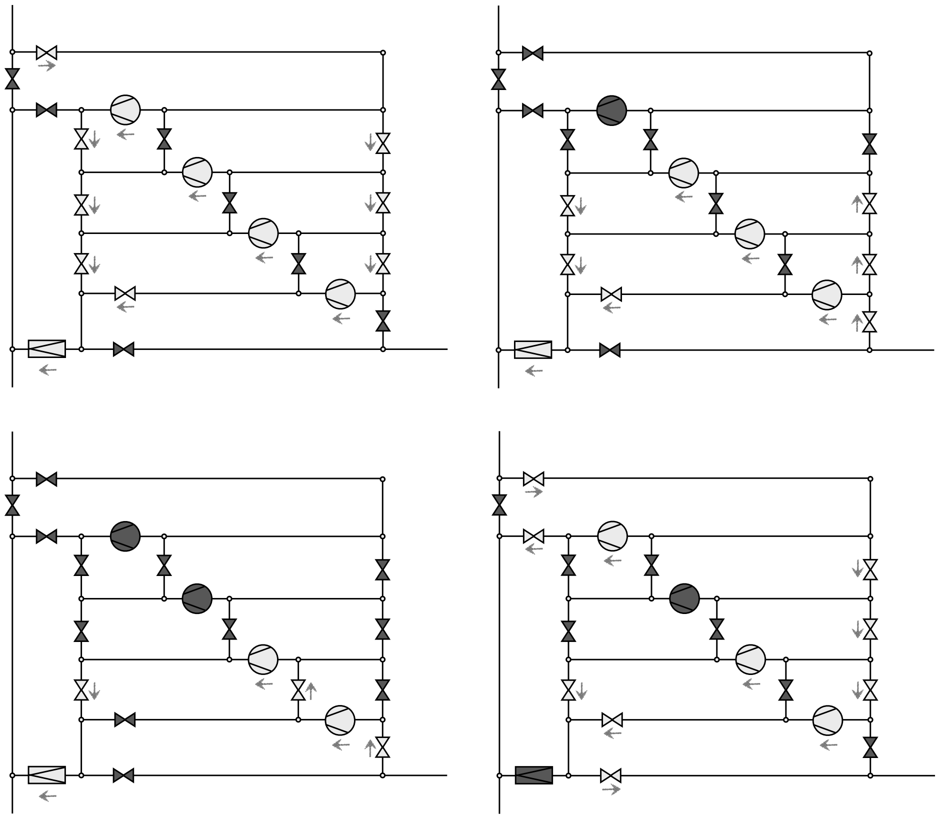


Modeled by

- Characteristic diagrams

- Highly nonlinear,
- no underlying PDE model.

Subnetwork Operation Modes



- Discrete Decisions cannot be taken independently,
- only reduced set allowed for each compressor station.

Mathematical approaches to validate nominations

Easy cases

Much is known about . . .

- Pure pipe networks,
- easy graphs, like paths and trees.

Methods

- Pure pipe networks: Can be transformed to a convex problem,
- Easy graphs: Dynamic Programming (see the survey by Carter).

What did we do?

Two stage approach

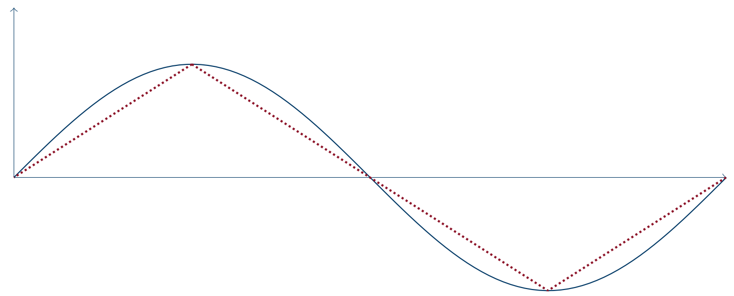
- First stage tries to compute discrete decisions (coarse models),
- Second stage checks these decisions (fine NLP model).

First stage

- Exact:
  - MILP-relaxation,
  - “special” MINLP-approach,
- Heuristic:
  - MPEC,
  - “reduced” NLP-approach + assignment heuristic.

Basis of our approach

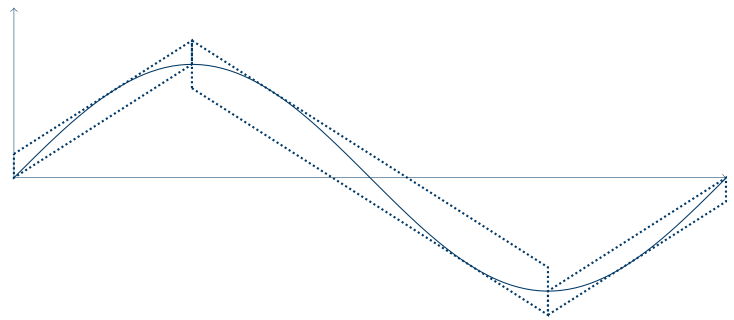
Classical: MIP approximations



- uses MIP formulations to approximate nonlinear constraints, e.g. incremental method or convex combination method.

Basis of our approach

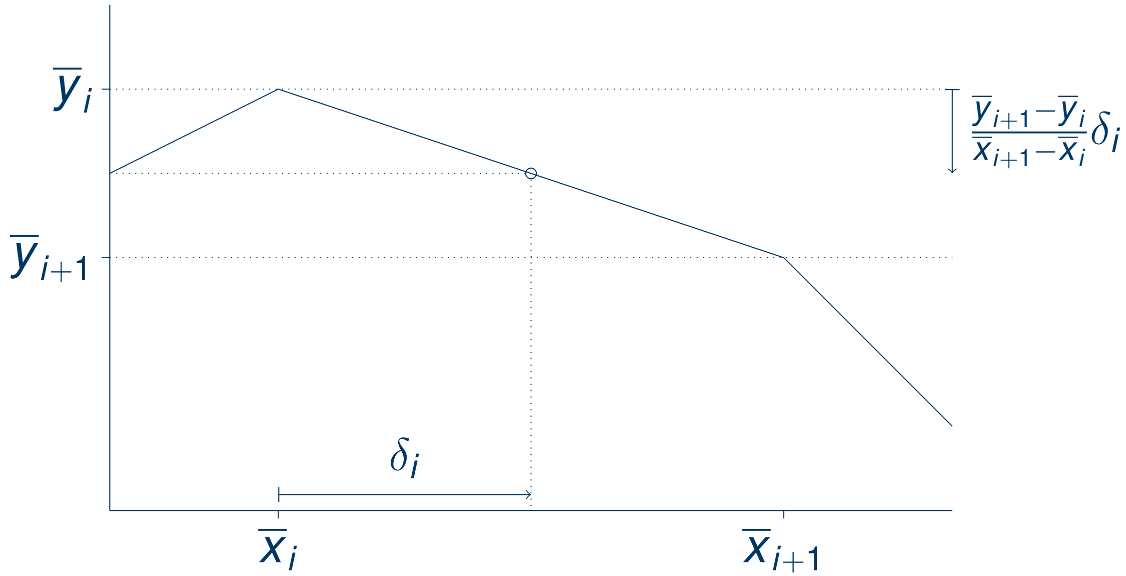
MIP relaxations



- uses MIP formulations to approximate nonlinear constraints, e.g. incremental method or convex combination method.
- modify classical formulations to incorporate over- and underestimators (Geißler, Martin, Morsi, LS; 2012)

Incremental Method: Basics

Markowitz, Manne (1957)



$$x = \overline{x}_0 + \sum_{i=1}^{n} \delta_i \qquad y = \overline{y}_0 + \sum_{i=1}^{n} \frac{\overline{y}_i - \overline{y}_{i-1}}{\overline{x}_i - \overline{x}_{i-1}} \delta_i$$

$$(\overline{x}_{i-1} - \overline{x}_{i-2}) z_{i-1} \leq \delta_i \quad \text{for all } i = 1, \dots, n.,$$

$$\delta_i \leq (\overline{x}_i - \overline{x}_{i-1}) z_i \quad \text{for all } i = 1, \dots, n-1.$$

Incremental method

Incorporating approximation errors (Geissler, Martin, Morsi, LS; 2012)

$$x = \overline{x}_0 + \sum_{i=1}^{n} \delta_i$$

$$y = \overline{y}_0 + \sum_{i=1}^{n} \frac{\overline{y}_i - \overline{y}_{i-1}}{\overline{x}_i - \overline{x}_{i-1}} \delta_i + e$$

$$\begin{aligned}
\epsilon_u^1(f) + \sum_{i=1}^{n-1} z_i(\epsilon_u^{i+1}(f) - \epsilon_u^i(f)) &\geq e \\
-\epsilon_o^1(f) - \sum_{i=1}^{n-1} z_i(\epsilon_o^{i+1}(f) - \epsilon_o^i(f)) &\leq e
\end{aligned}$$

$$\begin{aligned}
(\overline{x}_{i-1} - \overline{x}_{i-2})z_{i-1} \leq \delta_i &\quad \text{for all } i = 1, \ldots, n, \\
\delta_i \leq (\overline{x}_i - \overline{x}_{i-1})z_i &\quad \text{for all } i = 1, \ldots, n-1.
\end{aligned}$$

What can be computed now?

test set	$\|V\|$	$\|A_{\text{pi}}\|$	$\|A_{\text{sc}}\|$	$\|A_{\text{va}}\|$	$\|A_{\text{cv}}\|$	$\|A_{\text{cg}}\|$	# nom.
HN-AB	661	498	116	33	26	7	43
HN-SN	592	452	98	35	23	6	4227
gaslib-582	582	451	96	26	23	5	4227
gaslib-582-95	582	451	96	26	23	5	4227

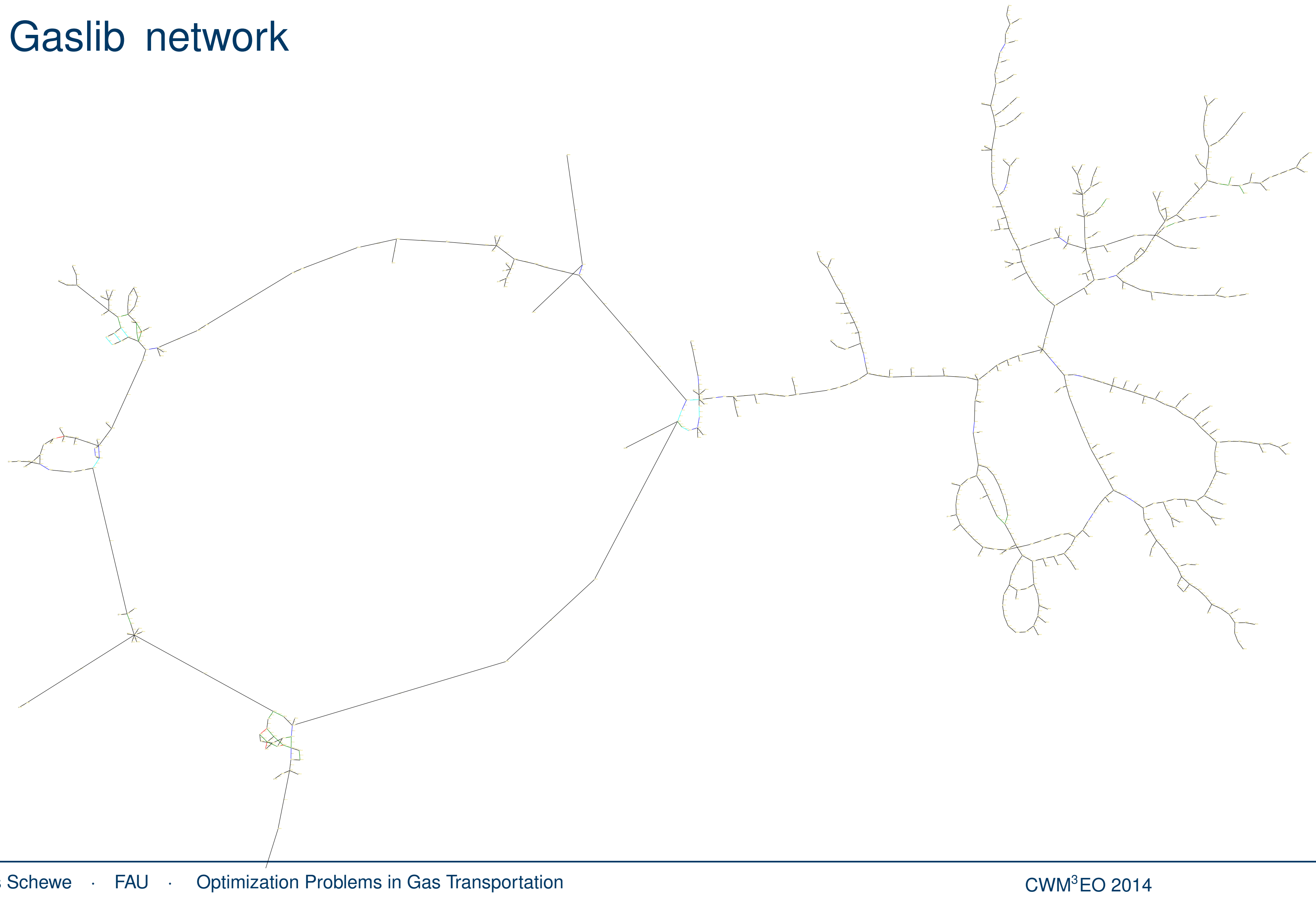
Public instances

gaslib-582 and gaslib-582-95 are available at:

http://gaslib.zib.de/

Impression

Gaslib network



Computational results – MILP approach

	slack 0	infeasible	slack > 0	no solution
HN-AB	33	3	7	0
HN-SN	3280	444	495	7
gaslib-582	2054	909	1230	34
gaslib-582-95	2831	716	661	19

- Timelimit: 14 400 s

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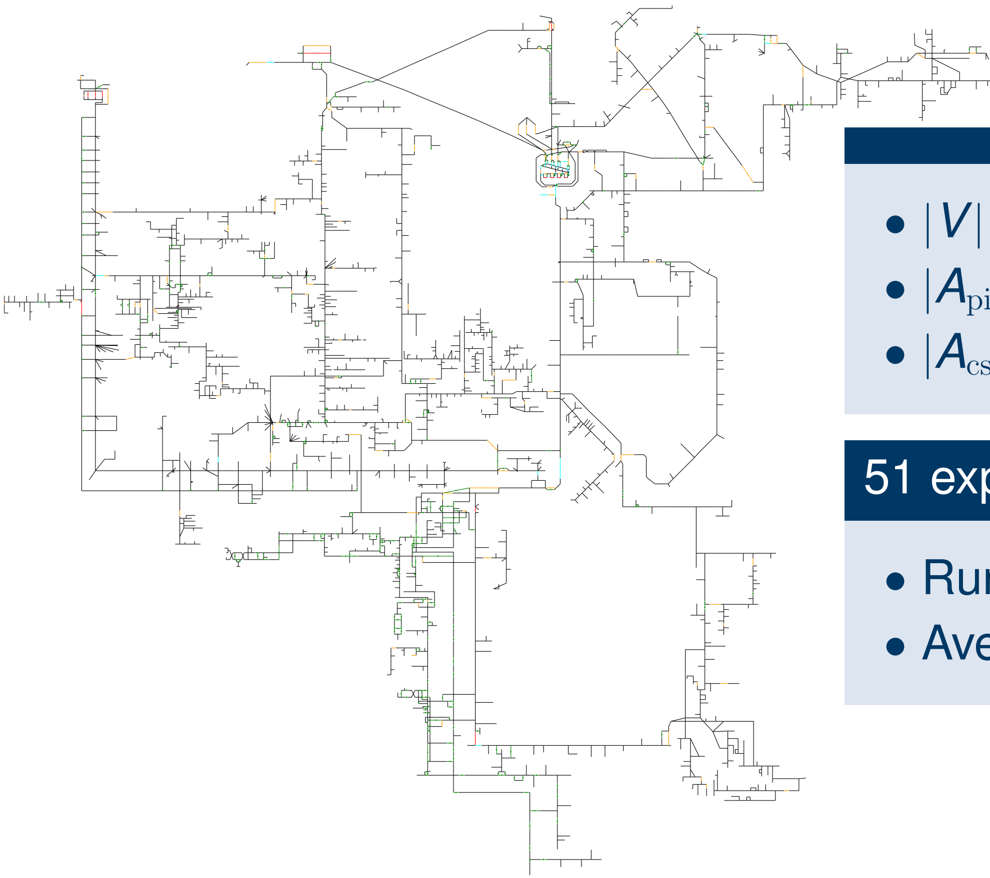
New variant of the MILP approach

under development



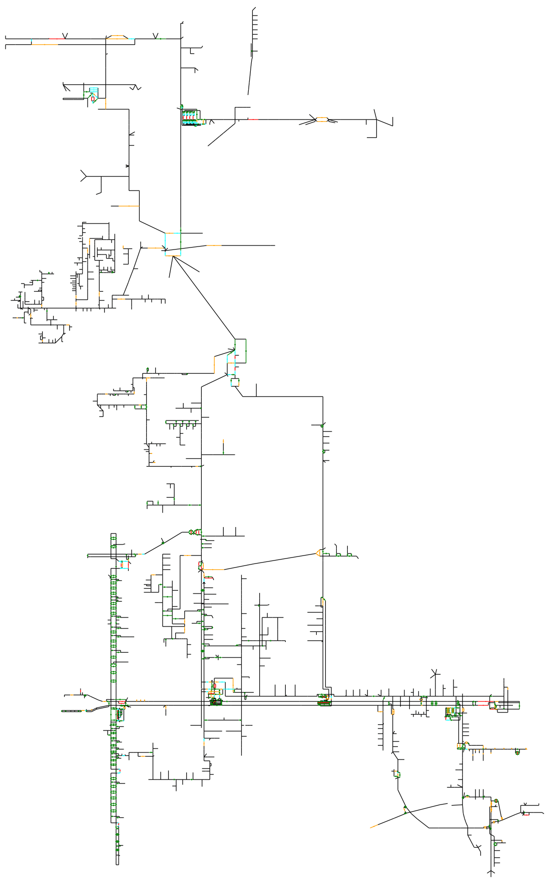
- $|V| = 4460$,
- $|A_{\mathrm{pi}}| = 3550$,
- $|A_{\mathrm{cs}}| = 12$.

51 expert instances

- Run time: 13 min to 810 min
- Average run time: 150 min

New variant of the MILP approach

under development



- $|V| = 2735$,
- $|A_{\mathrm{pi}}| = 3074$,
- $|A_{\mathrm{cs}}| = 41$.

29 expert instances

- Run time: 3 h to 49 h
- Average run time: 17 h

Future directions

Problems

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There is much to do

Better physics

- Transient, non-isothermal,
- Gas mixtures,
- reach “simulation accuracy”.

More economy

- Booking validation,
- Technical capacities,
- Network extension planning.

Overall

- Faster,
- larger networks.

Next up . . .

DFG CRC/TR 154

Mathematical modelling, simulation and optimization using the example of gas networks

Advertisement: Our book

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