Applied Math Days
Budapest University of Technology and Economics
October 26, 2015

Alfio Quarteroni

Mathematical models and their impact on our daily life

EPFL, Lausanne (Switzerland)

POLITECNICO of MILAN (on leave)

POLITECNICO DI MILANO

MOX

CMCS

MATHICSE

EPFL ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE

Basic Definitions - Basic Concepts

Modeling

Using equations to describe physical, social, environmental... processes

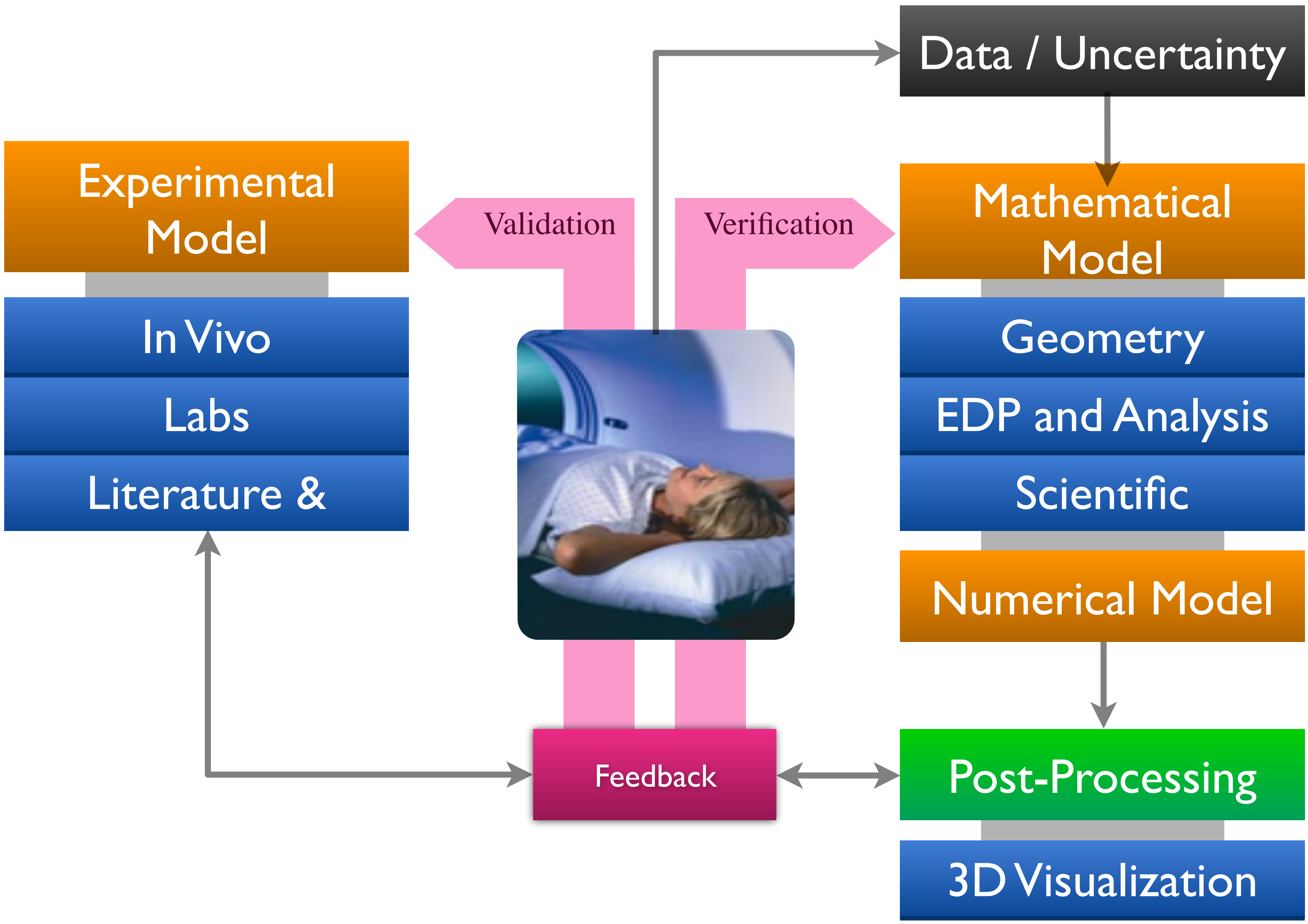
Multiphysics Problems

Described by models made of at least two different constituents that describe different kind of physics -- often leading to heterogeneous (different kind of) PDEs

Multiscale Problems

Solution components to the mathematical model have multiple features at multiple scales (temporal and / or spatial)
(e.g. different description levels in material science, evolution under different dynamics in different time intervals and co-existence of different spatial gradients in fluid dynamics, combination of different geometrical dimensions in highly structured networks, etc.)

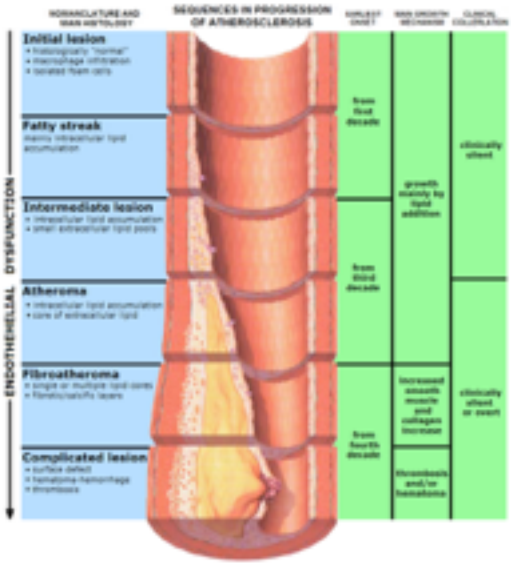
Mathematical Modeling and Simulation: an Outlook

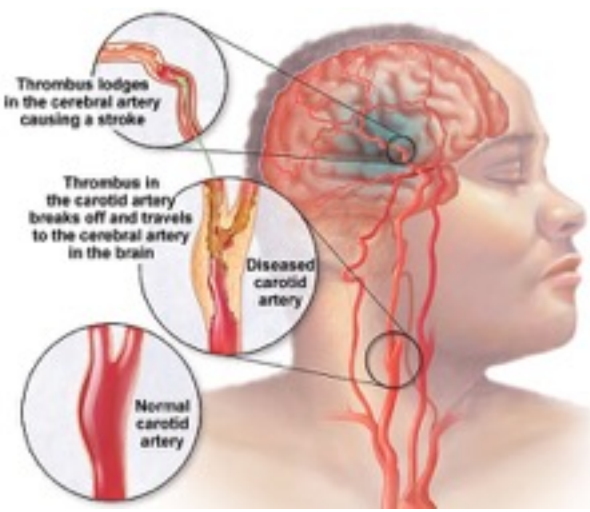


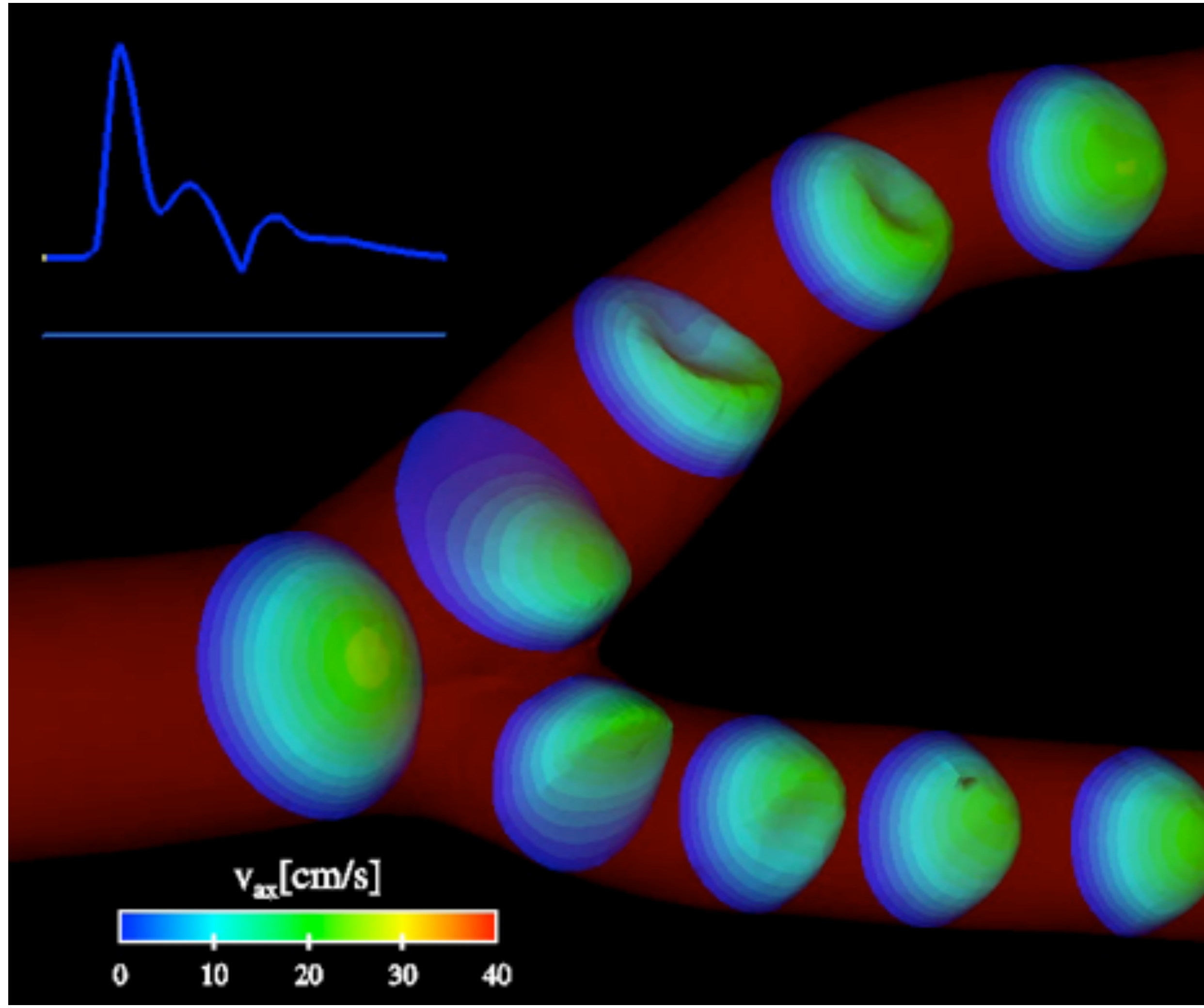
Mathematics
for our health...

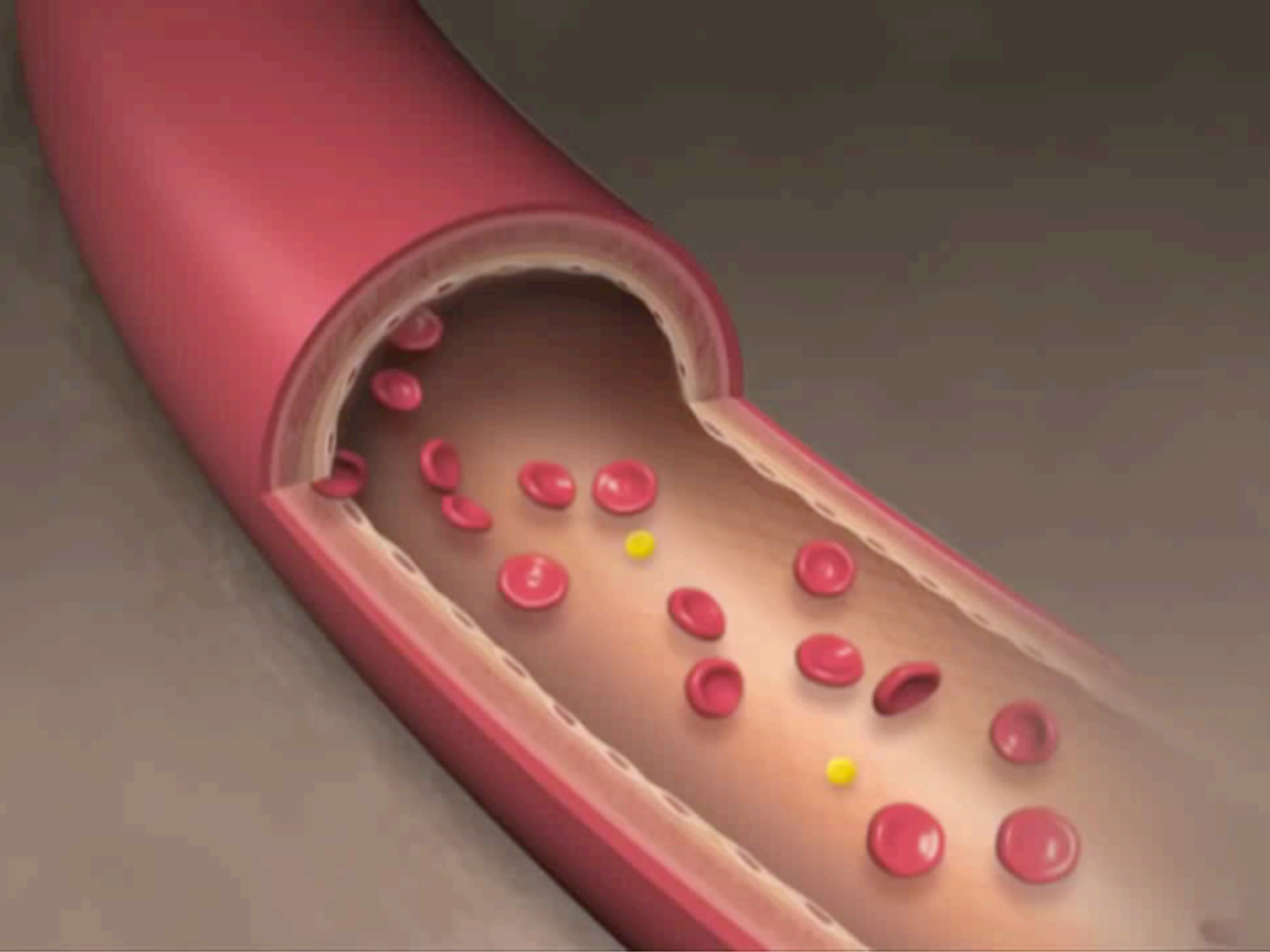
Local flow analysis - Rigid walls

- Navier-Stokes equations, Newtonian pulsatile flow in large vessels

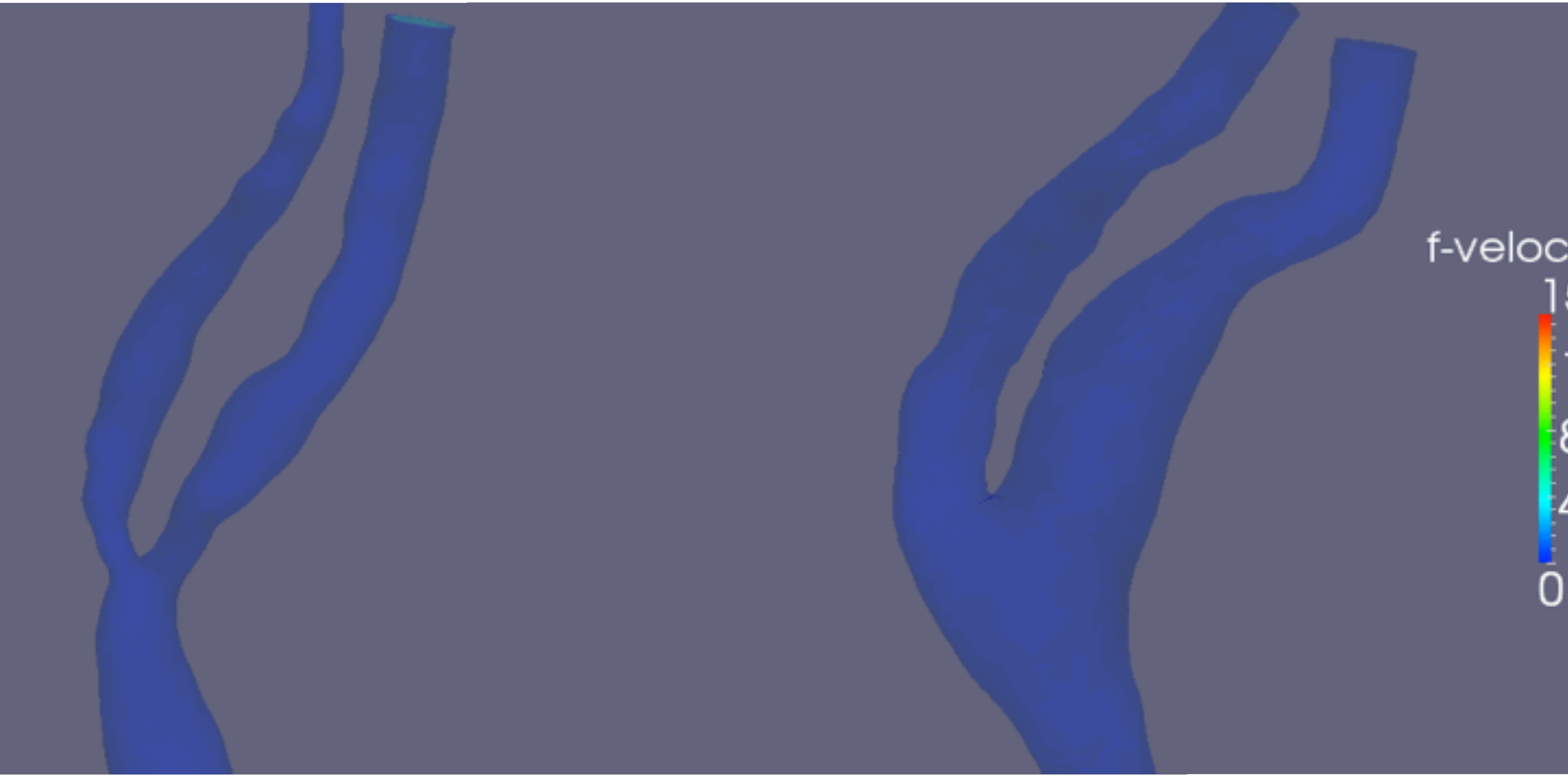








"Optimal" surgery for carotid arteries



Before TEA **After TEA**

(acknowledgment: Fondazione Cà Granda, Policlinico di Milano; E.Faggiano, C.Vergara, MOX)

Local Flow Analysis: Fluid-Structure-Interaction (FSI)

Equations for the geometry (harmonic or elastic extension of wall displacement):

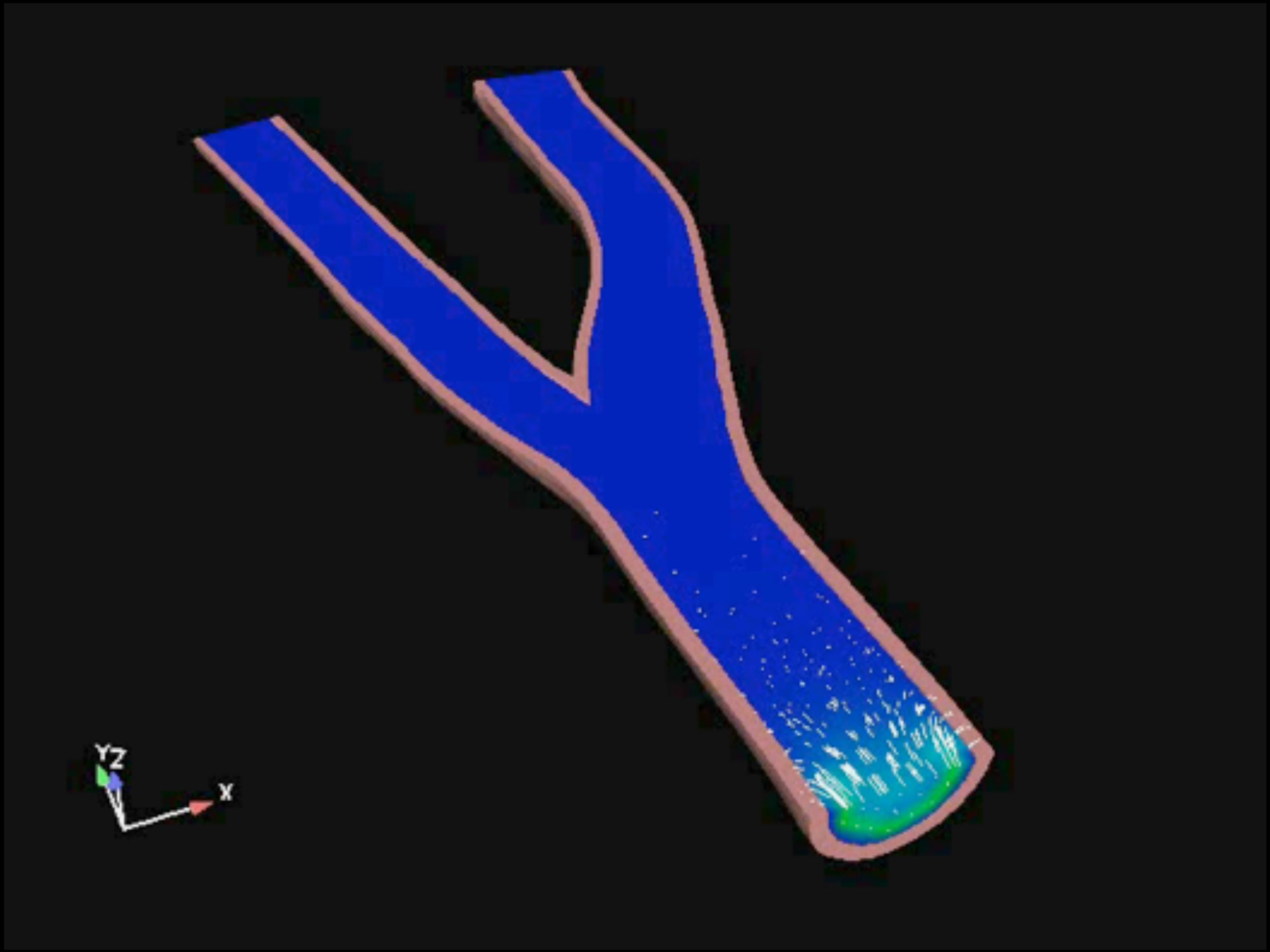
$$\hat{\eta}_f = \mathrm{Ext}(\hat{\eta}_{s|\hat{\Gamma}}), \quad \hat{\mathbf{w}} = \frac{\partial \hat{\eta}_f}{\partial t}, \quad \Omega_f(t) = (I + \hat{\eta}_f)(\hat{\Omega}_f)$$

Equations for the fluid (Navier-Stokes equations, ALE frame of reference):

$$\begin{aligned}
\frac{\rho_f}{J_{\hat{A}}} \frac{\partial J_{\hat{A}} \mathbf{u}_f}{\partial t}\Big|_{\hat{\mathbf{x}}} + \mathrm{div}(\rho_f \mathbf{u}_f \otimes (\mathbf{u}_f - \mathbf{w}) - \sigma_f(\mathbf{u}_f, P)) = 0, &\quad \text{in} \quad \Omega_f(t) \\
\mathrm{div}\, \mathbf{u}_f = 0, &\quad \text{in} \quad \Omega_f(t) \\
\mathbf{u}_f = \mathbf{u}_D, &\quad \text{on} \quad \Gamma_{f,D} \\
\sigma_f(\mathbf{u}_f, P)\mathbf{n}_f = \mathbf{g}_{f,N}, &\quad \text{on} \quad \Gamma_{f,N} \\
\mathbf{u}_f = \mathbf{w}, &\quad \text{on} \quad \Gamma(t)
\end{aligned}$$

Equations for the structure (hyperelastic materials):

$$\begin{aligned}
\hat{\rho}_{s,0} \frac{\partial^2 \hat{\eta}_s}{\partial t^2} - \mathrm{div}_{\hat{\mathbf{x}}}(\hat{\mathbf{F}}_s \hat{\Sigma}) = 0, &\quad \text{in} \quad \hat{\Omega}_s \\
\hat{\eta}_s = 0 &\quad \text{on} \quad \hat{\Gamma}_{s,D} \\
\hat{\mathbf{F}}_s \hat{\Sigma} \hat{\mathbf{n}}_s = \hat{J}_s |\hat{\mathbf{F}}_s^{-T} \hat{\mathbf{n}}_s| \hat{\mathbf{g}}_{s,N}, &\quad \text{on} \quad \hat{\Gamma}_{s,N} \\
\hat{\mathbf{F}}_s \hat{\Sigma} \hat{\mathbf{n}}_s = \hat{J}_s \hat{\sigma}_f(\mathbf{u}_f, P) \hat{\mathbf{F}}_s^{-T} \hat{\mathbf{n}}_s, &\quad \text{on} \quad \hat{\Gamma}
\end{aligned}$$



Numerical simulation: G.Fourestey

"How do we solve these equations?"

A long story to tell…

Projection methods

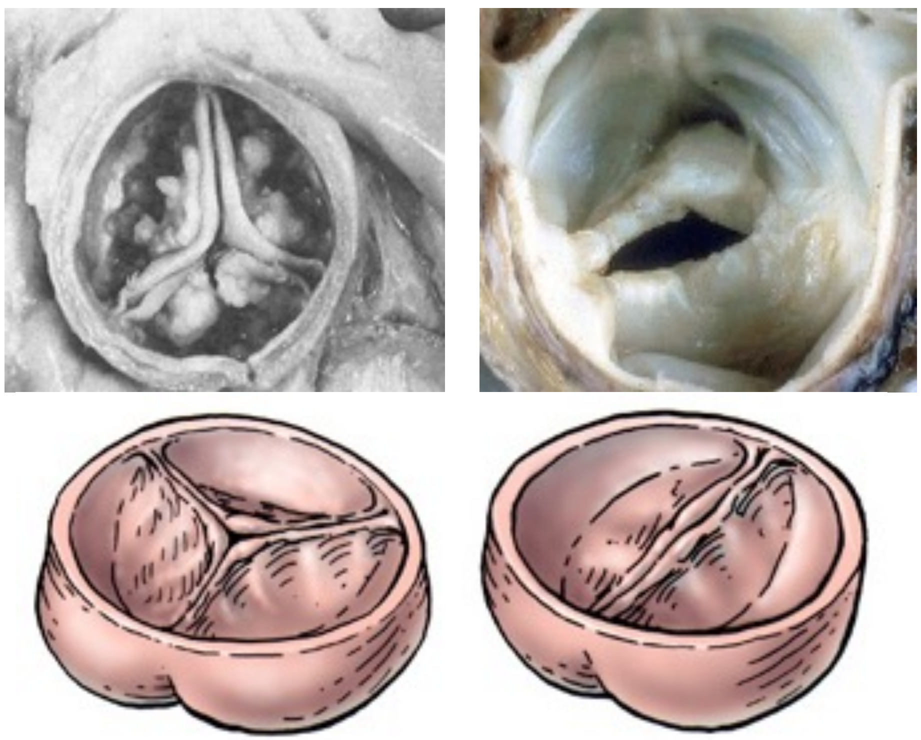
consistency stability convergence

accuracy

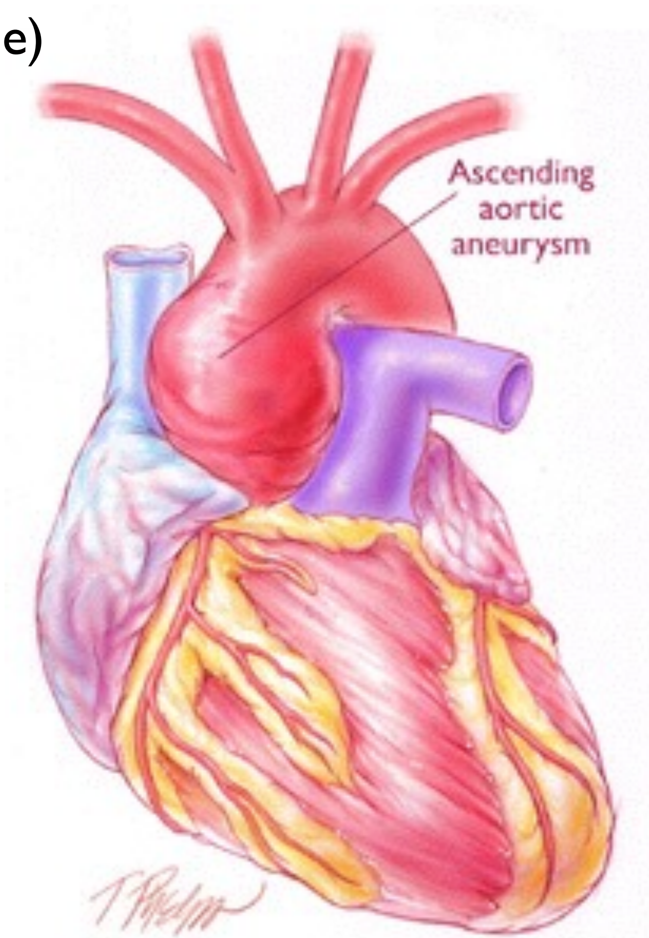
complexity

Jet flow downstream bicuspid aortic valves

Formazione di aneurismi in aorta ascendente in presenza di valvole aortiche bicuspidi (BAV) (1-2% della popolazione)



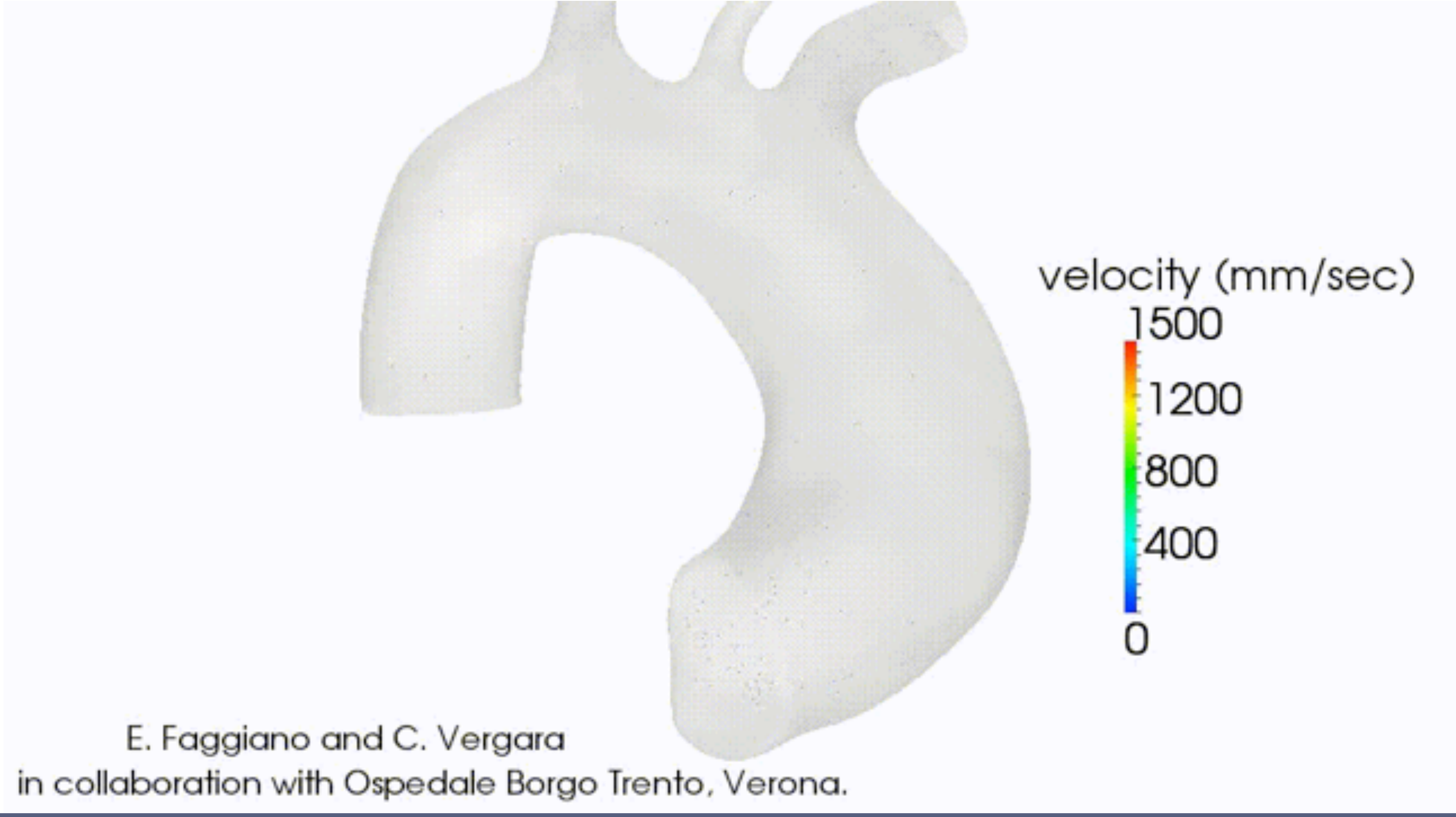
Tricuspid valve Bicuspid valve



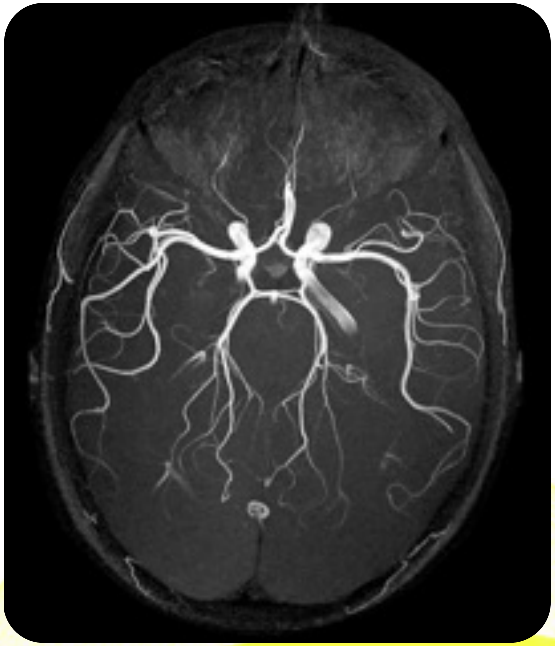
La presenza di BAV nel 40-60% dei pazienti si accompagna ad una maggior propensione alla dilatazione aortica e alla formazione di aneurismi

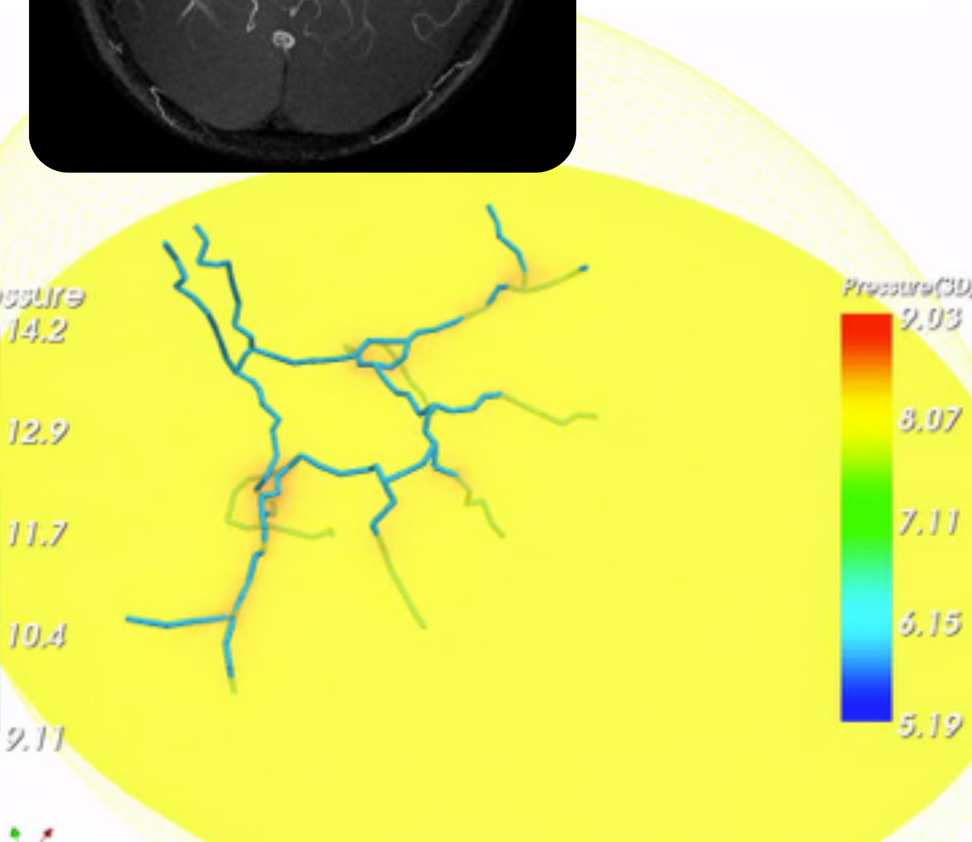
COSA PUO' FARE LA MATEMATICA

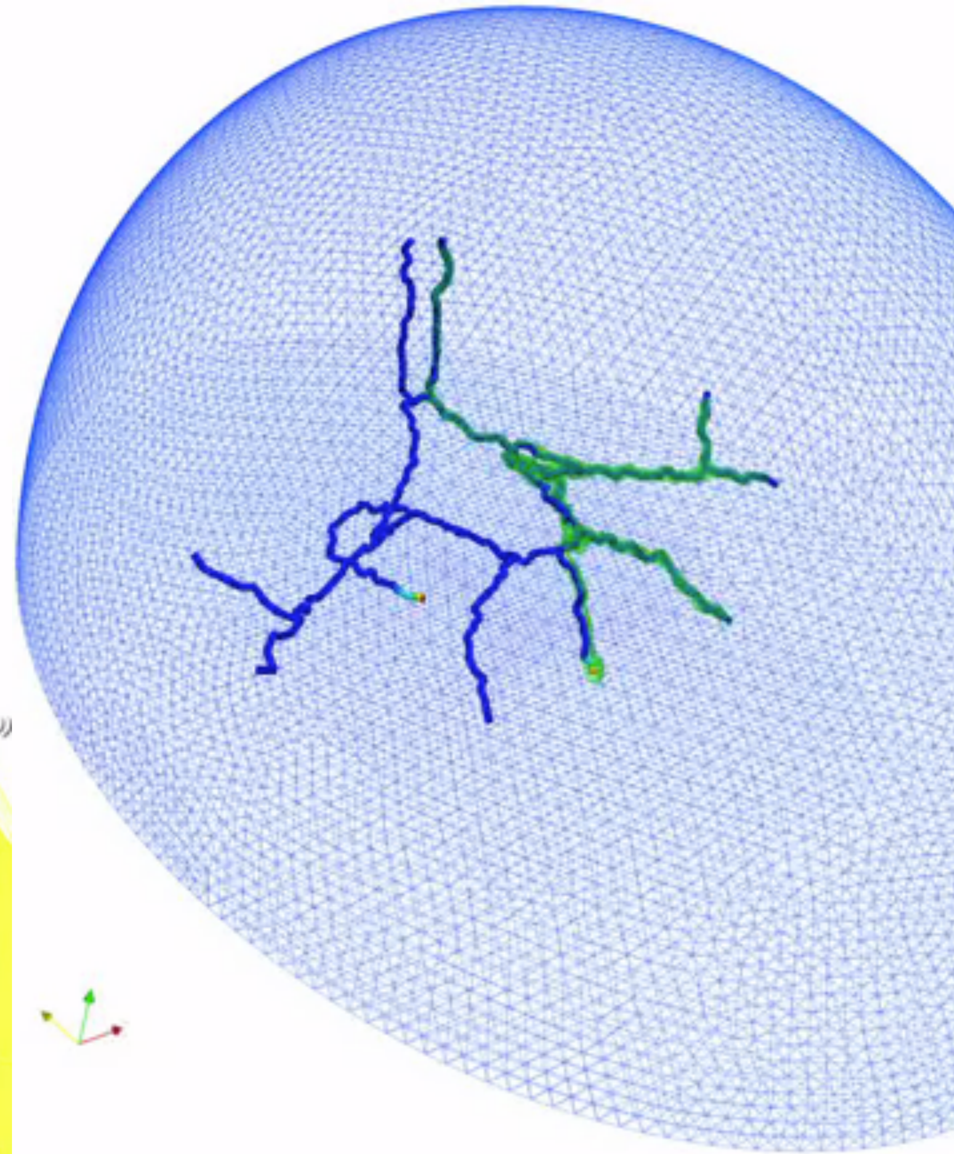
Il modello matematico simula la presenza di un getto che investe la parete esterna dell'arco ascendente dove tipicamente si sviluppano aneurismi



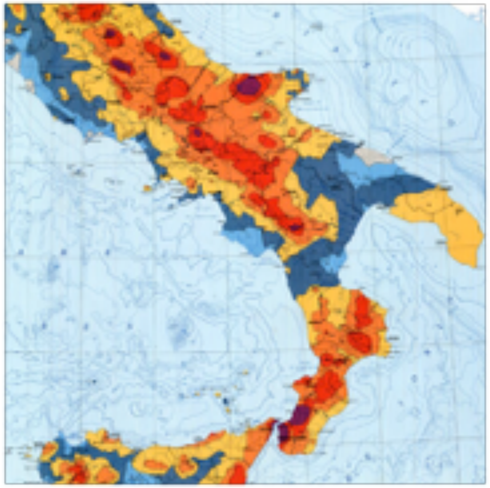
Oxygen distribution in the brain







Mathematics for our Safety

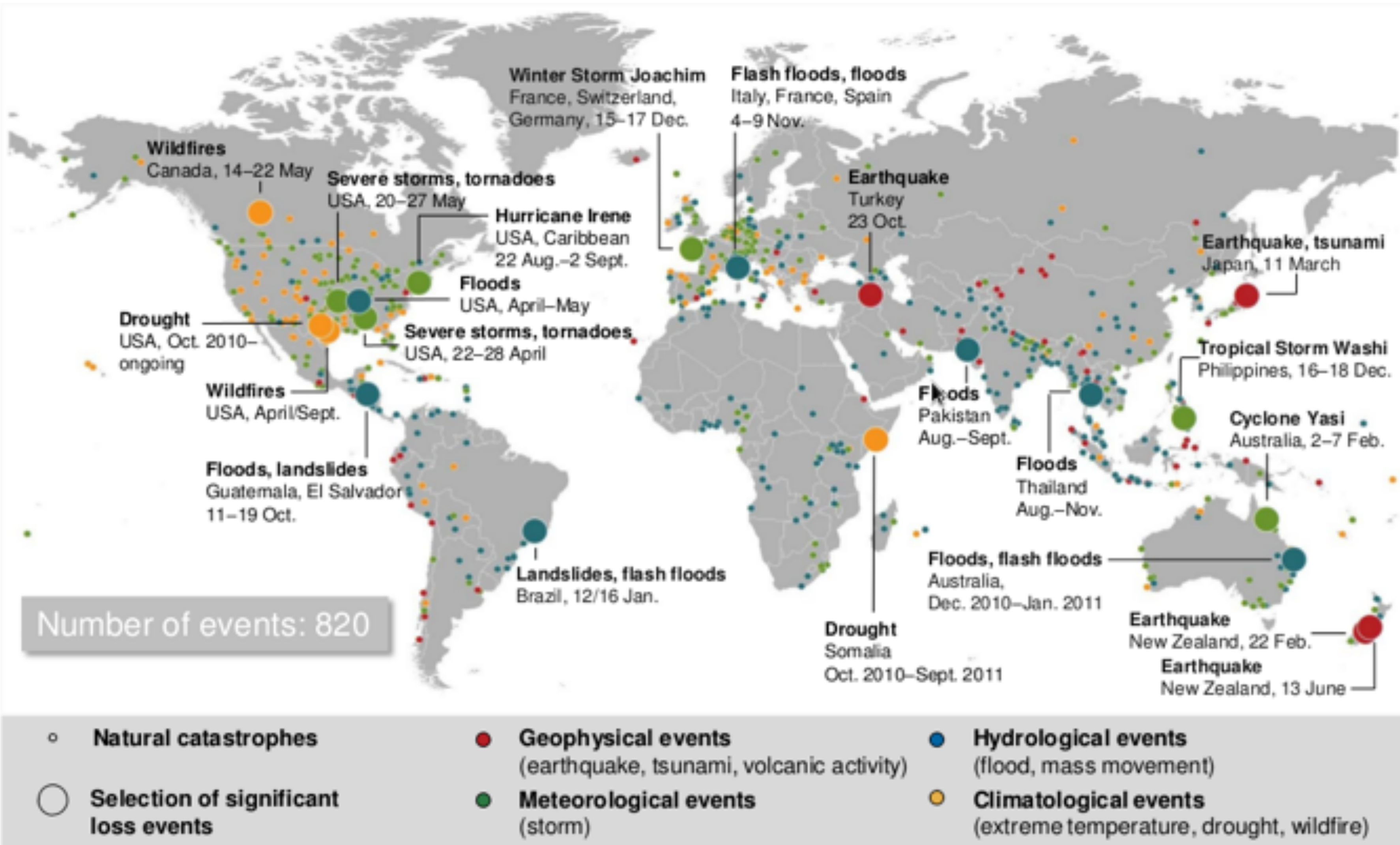


Significant Natural Catastrophes 1980 - 2011
10 costliest earthquake events worldwide ordered by insured losses

Period	Event	Affected Area	Overall Losses	Insured Losses	Fatalities
			US$ m, Original Values		
11.3.2011	Earthquake, tsunami	Japan: Honshu, Aomori, Tohoku; Miyagi, Sendai; Fukushima, Mito; Ibaraki; Tochigi, Utsunomiya	210,000	35,000-40,000	15,840
17.1.1994	Earthquake	USA: CA, Northridge, Los Angeles, San Fernando Valley, Ventura, Orange	44,000	15,300	61
22.2.2011	Earthquake	New Zealand: South Island, Canterbury, Christchurch, Lyttelton	18,000	15,000	185
27.2.2010	Earthquake, tsunami	Chile: Bió Bió, Concepción, Talcahuano, Coronel, Dichato, Chillán; Del Maule, Talca, Curicó	30,000	8,000	520
4.9.2010	Earthquake	New Zealand: Canterbury, Christchurch, Avonside, Omihi, Timaru, Kaiapoi, Lyttelton	7,500	6,000	
17.1.1995	Earthquake	Japan: Hyogo, Kobe, Osaka, Kyoto	100,000	3,000	6,430
13.6.2011	Earthquake	New Zealand: Canterbury, Christchurch, Lyttelton	3,500	2,500	1
26.12.2004	Earthquake, tsunami	Sri Lanka, Indonesia, Thailand, India, Bangladesh, Myanmar, Maldives, Malaysia	10,000	1,000	220,000
17.10.1989	Earthquake	USA: CA, Loma Prieta, Santa Cruz, San Francisco, Oakland Berkeley, Silicon Valley	10,000	960	68
23.10.2001	Earthquake	Japan: Honshu, Niigata, Ojiya, Tokyo, Nagaoka, Yamakoshi	28,000	760	46

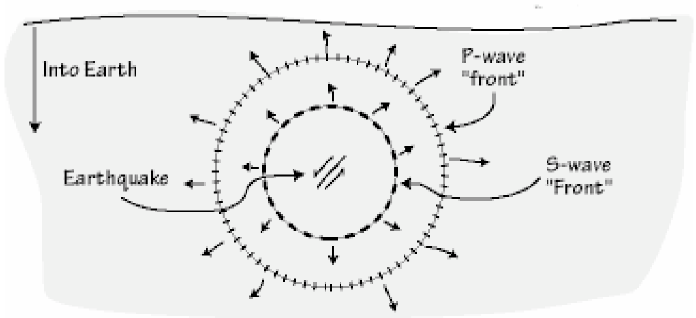
Courtesy of M.Stupazzini - MunichRe

Natural Catastrophes 2011
World map

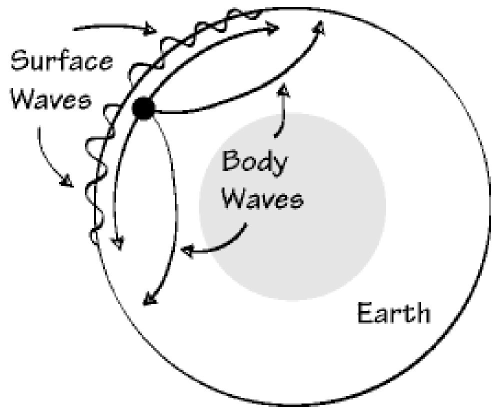


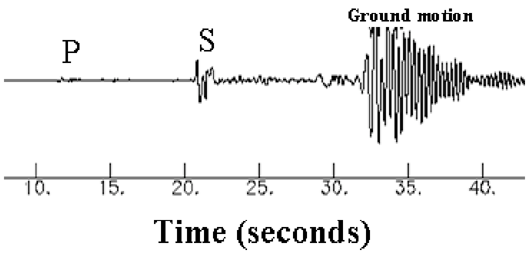
Seismic waves propagate vibrations that carry energy from the source of the shaking outward in all directions

- There are different kind of seismic waves. The most important ones are
  - Compressional or P (primary)
  - Transverse or S (secondary)
  - Love
  - Rayleigh



- An earthquake radiates P and S waves in all directions.
- The interaction of the P and S waves with Earth's surface $\rightarrow$ surface waves.
- P and S waves travel at different speeds (used to locate earthquakes)



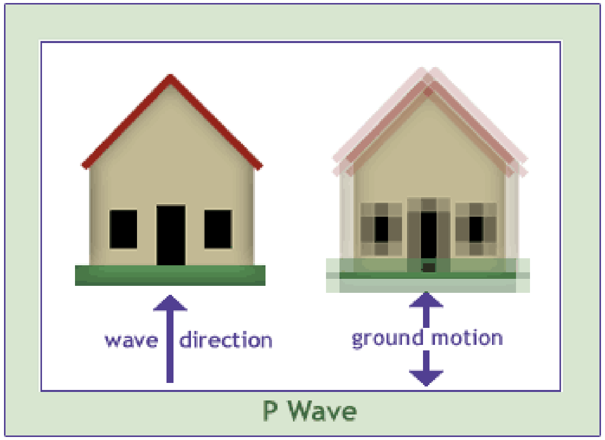


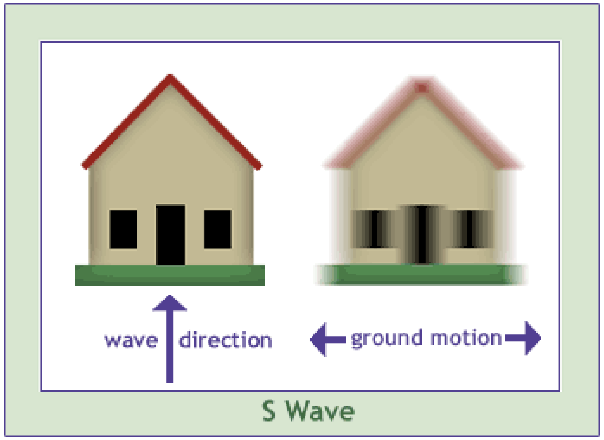
P-Waves (Compressional)

- The ground is vibrated in the direction the wave is propagating.
- Travel through all types of media.
- $C_P = \sqrt{(\lambda + 2\mu)/\rho}$ (P-wave velocity)
- Typical speed: $\sim 1 \rightarrow 14$ km/sec

S-Waves (Transverse)

- The ground is vibrated in the perpendicular direction to that the wave is propagating.
- Travel only through solid media.
- $C_S = \sqrt{\mu/\rho}$ (S-wave velocity)
- Typical speed: $\sim 1 \rightarrow 8$ km/sec





Much of the damage close to an earthquake is the result of strong shaking caused by S-waves.

The Mathematical Model (with I.Mazzieri and P.Antonietti)

Equilibrium equations for an elastic bounded medium subjected to an external force $\mathbf{f}$

$$\begin{cases} \rho\partial_{tt}\mathbf{u} - \nabla\cdot\underline{\sigma}(\mathbf{u}) + 2\rho\zeta\,\partial_t\mathbf{u} + \rho\zeta^2\mathbf{u} = \mathbf{f} & \text{in } \Omega\times(0,T], \\ + \text{ B.C.s} & \text{on } \partial\Omega\times[0,T], \\ + \text{ I.C.s} & \text{in } \Omega\times\{0\}. \end{cases}$$

- $\mathbf{u}$ displacement of the medium
- $\underline{\epsilon}(\mathbf{u}) = \frac{1}{2}(\nabla\mathbf{u} + \nabla\mathbf{u}^\top)$ strain tensor
- $\underline{\sigma}(\mathbf{u}) = \lambda\nabla\cdot\mathbf{u}\,\underline{I} + 2\mu\underline{\epsilon}(\mathbf{u}) = \underline{\underline{\mathbf{D}}}\,\underline{\epsilon}(\mathbf{u})$ stress tensor
- ρ material density, λ, μ Lamé elastic coefficients
- $2\rho\zeta\,\partial_t\mathbf{u} + \rho\zeta^2\mathbf{u}$ viscous forces

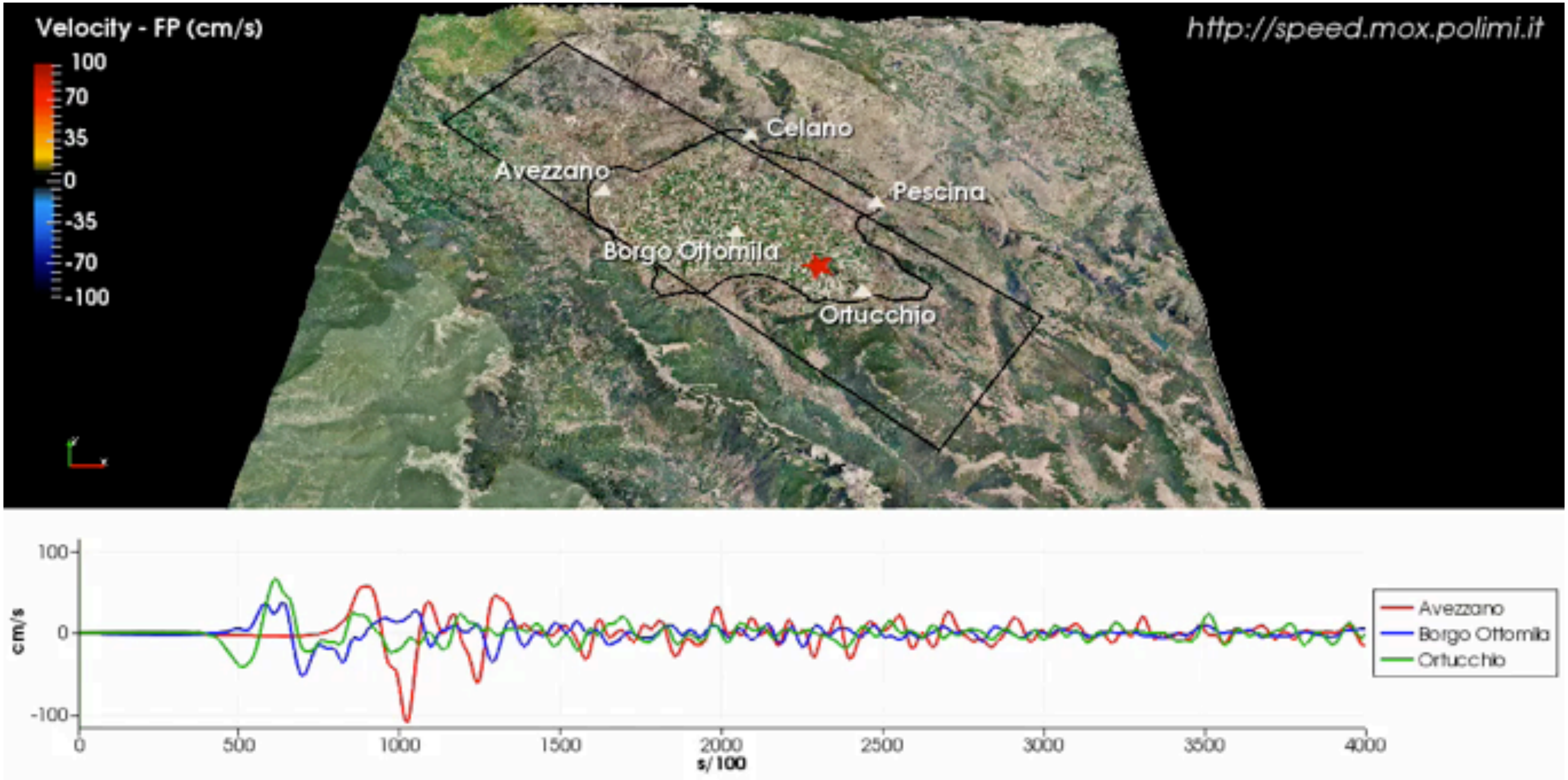
In applications the decay factor ζ is typically $\approx 0.01s^{-1}$

Approximation principles: Domain Decomposition fitting material properties - Discontinuous Galerkin treatment of interfaces; high order spectral elements within each subdomain

- Accurate description of the behaviour at interfaces
  $\Longrightarrow$ high-order accuracy
- Waves are typically propagated over many periods
  $\Longrightarrow$ control the numerical dispersion and dissipation errors
- Complex geometries and strong contrasts in wave speeds
  $\Longrightarrow$ methods that accommodate non-conforming grids
- The size of the bodies excited is large relative to the wavelengths of interest
  $\Longrightarrow$ efficient and scalable implementation

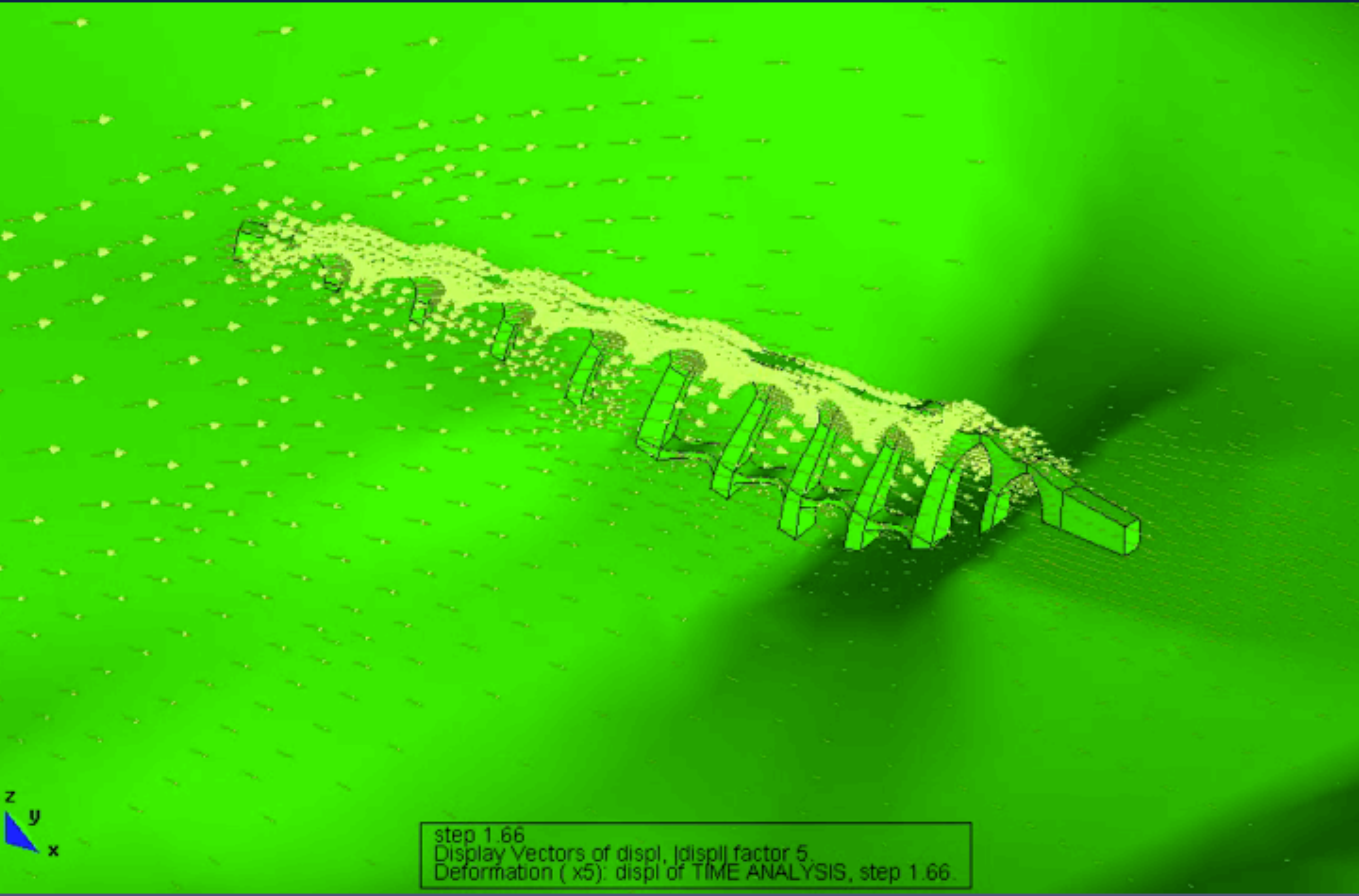
Marsica Earthquake

(13 January 1915 - magnitudo 7.0 - 32000 casualties)

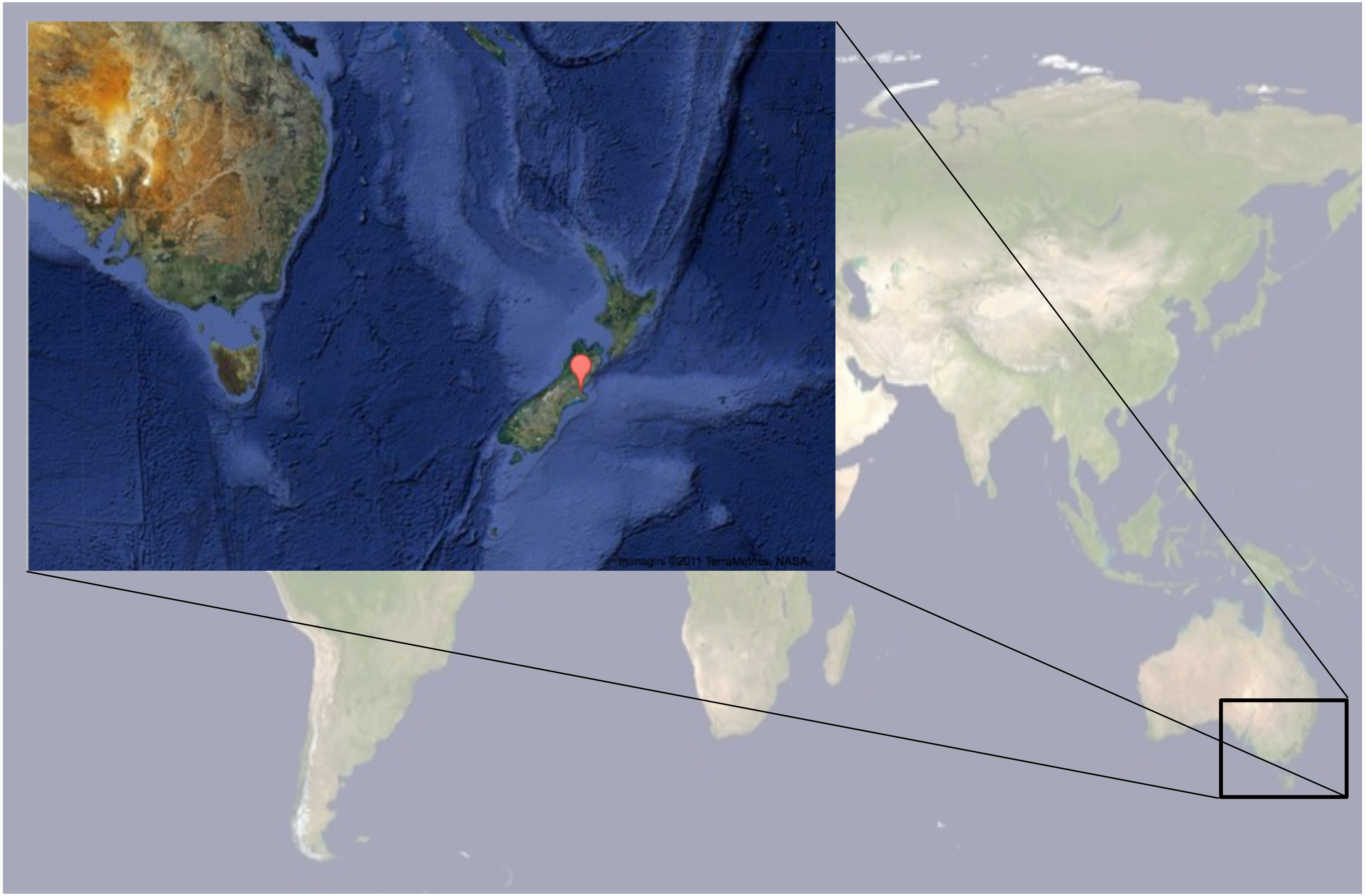


Acquasanta Bridge, Liguria, Italy

Acquasanta Bridge, Liguria, Italy



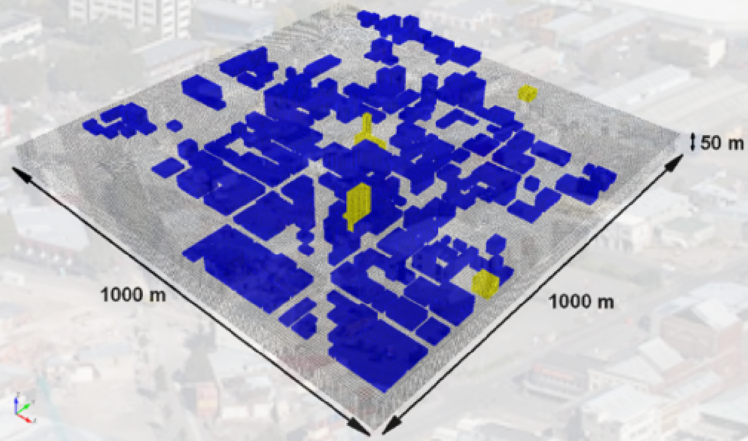
Christ Church, Nuova Zelanda (6.2 Richter , 22.02.2011)

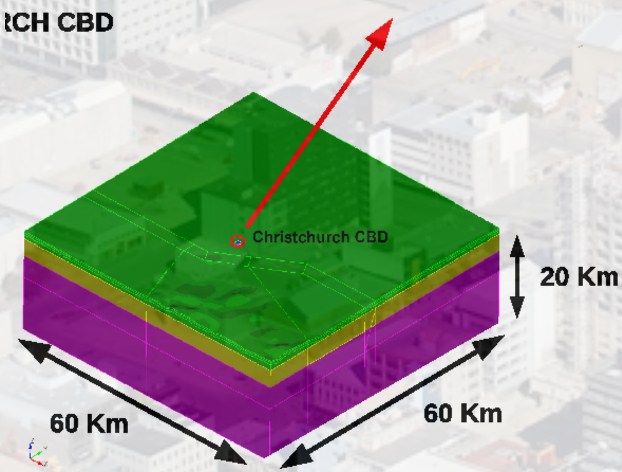


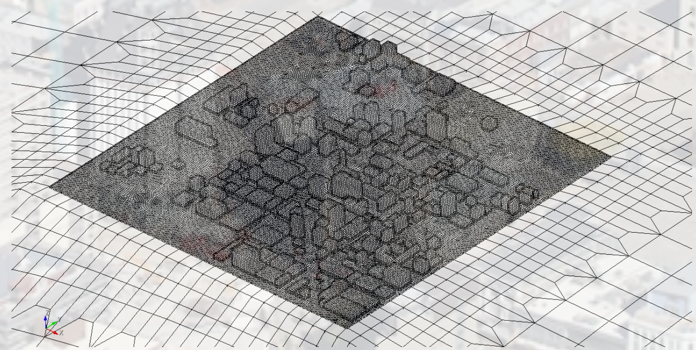
Christ Church Business District

Christ Church, New Zealand (6.2 Richter - 22 February 2011)

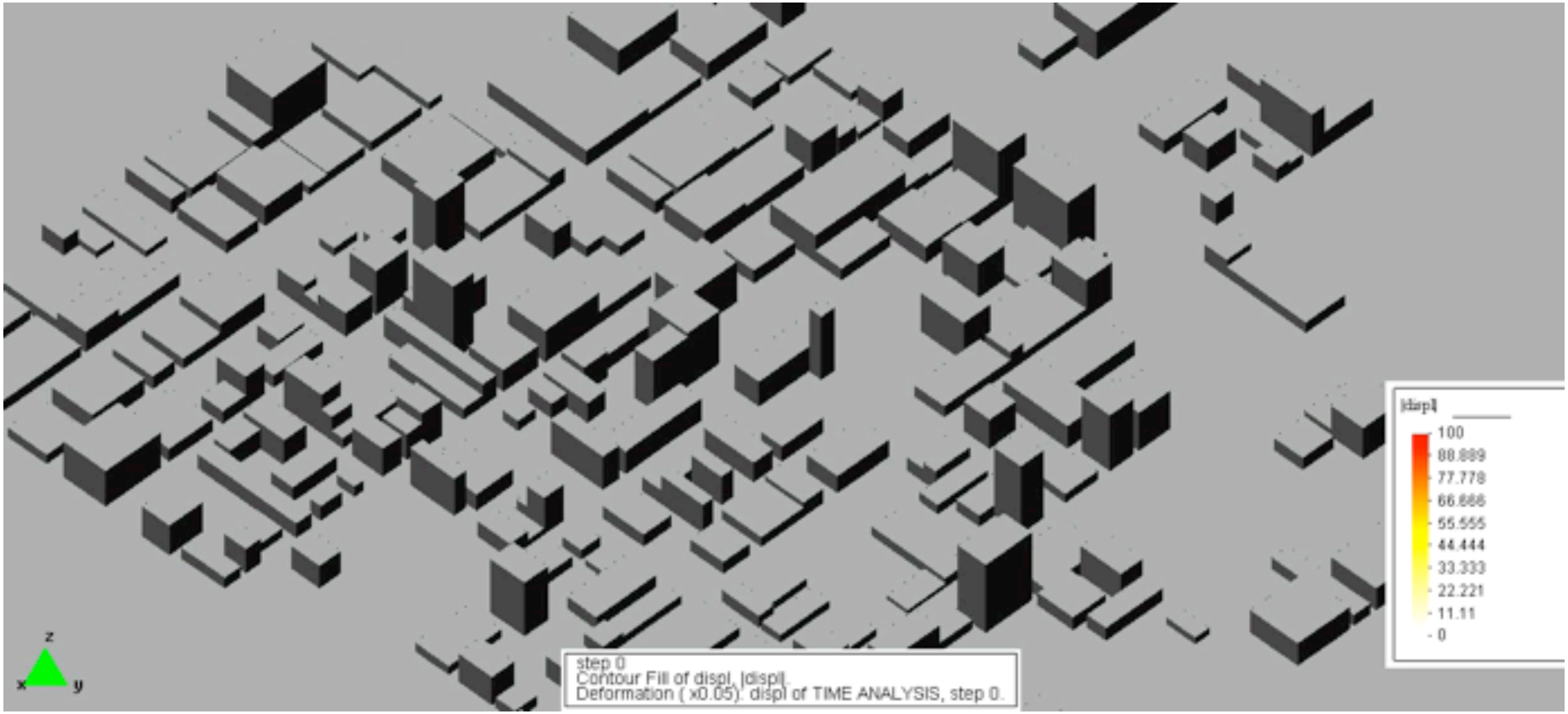
Christchurch Central Business District



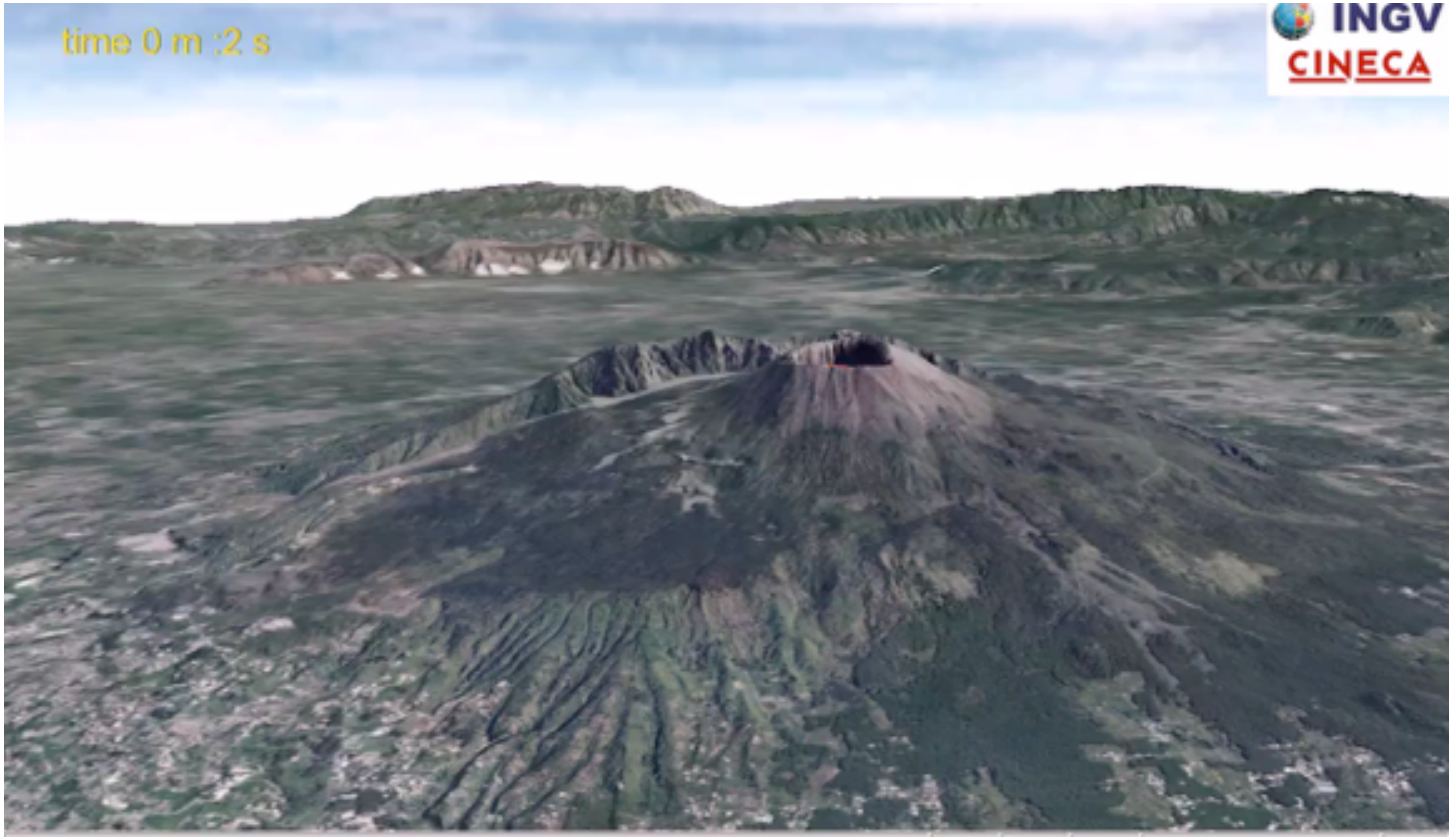




Christchurch Central Business District

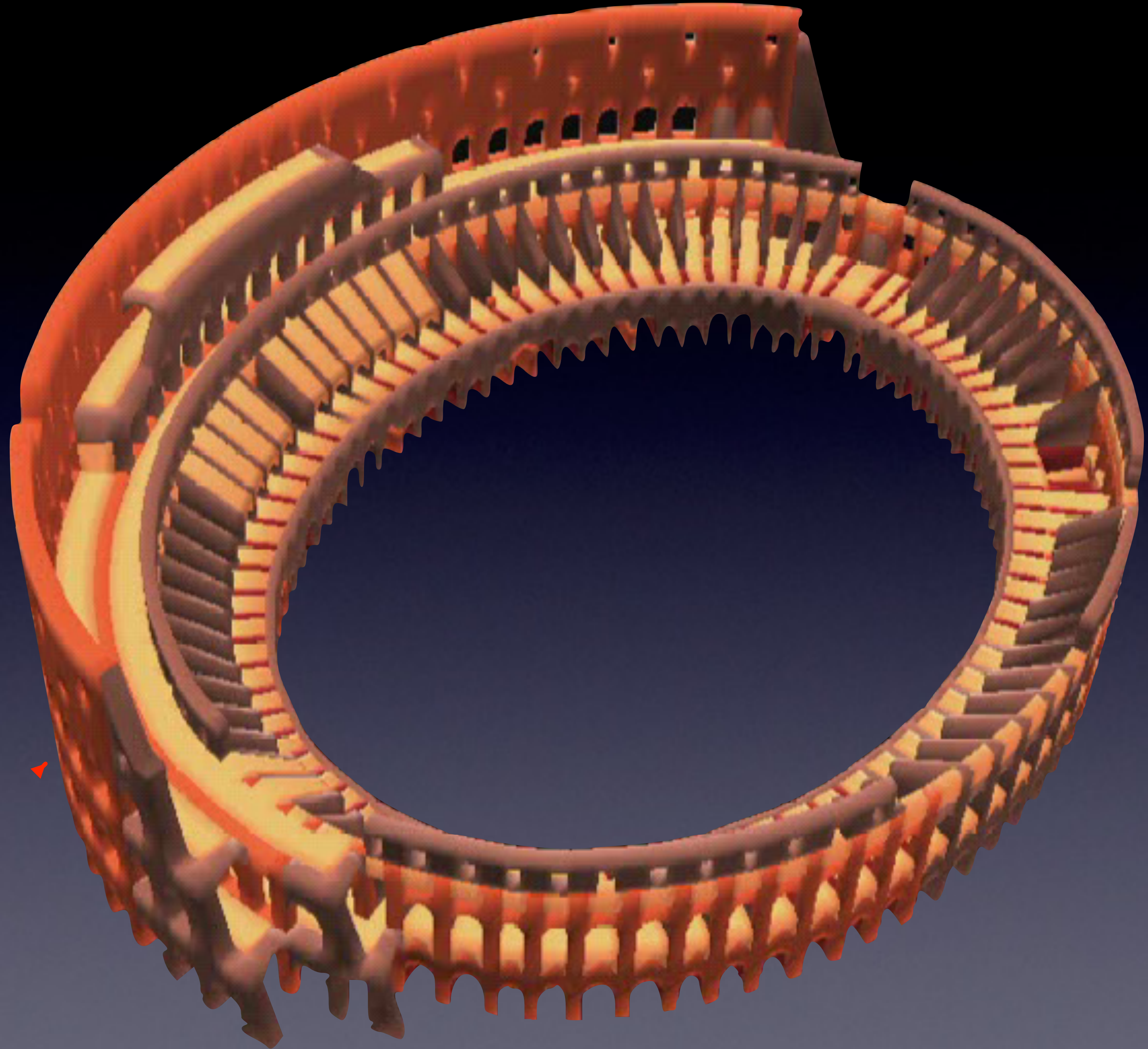


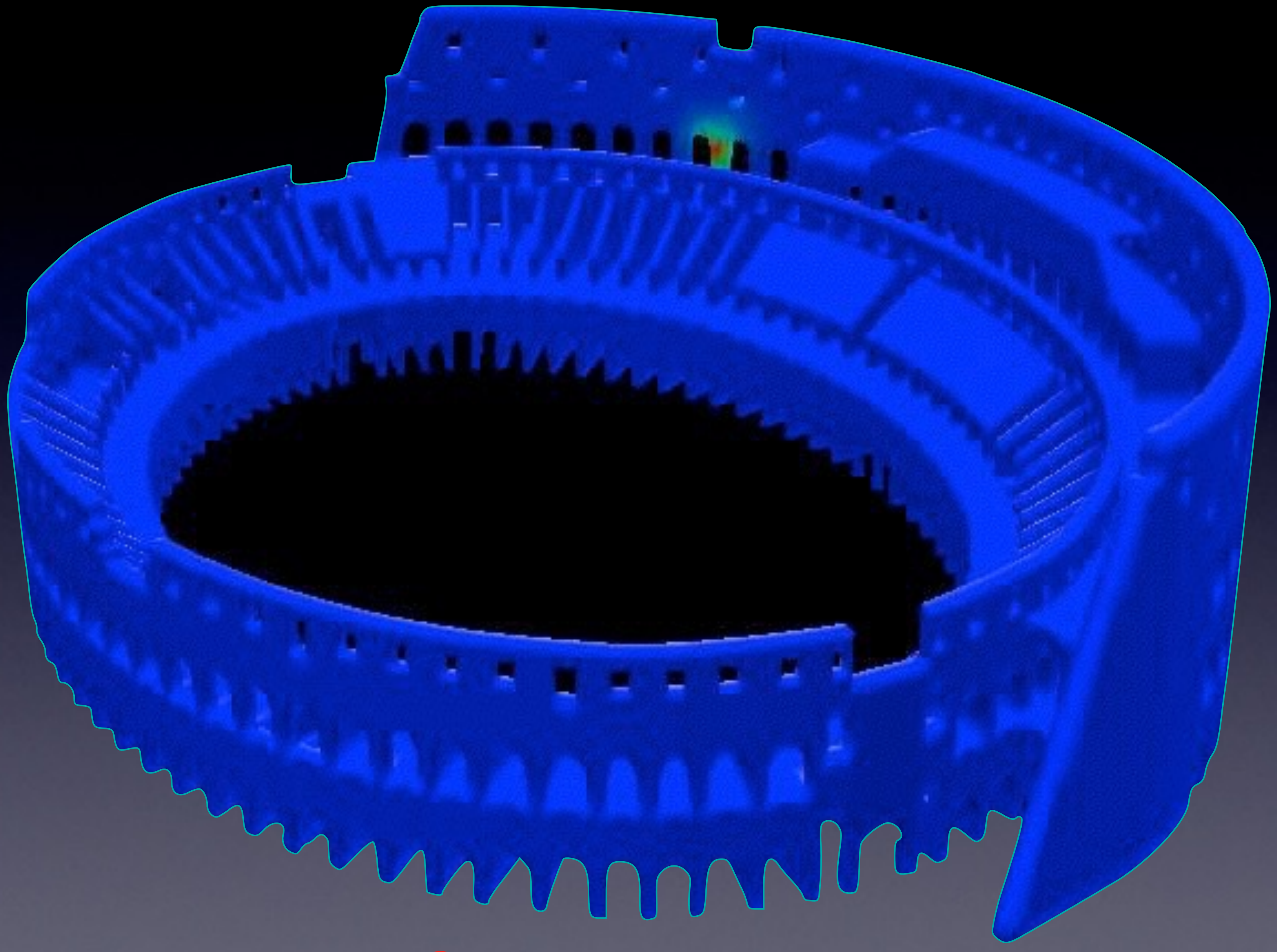
Volcanoes



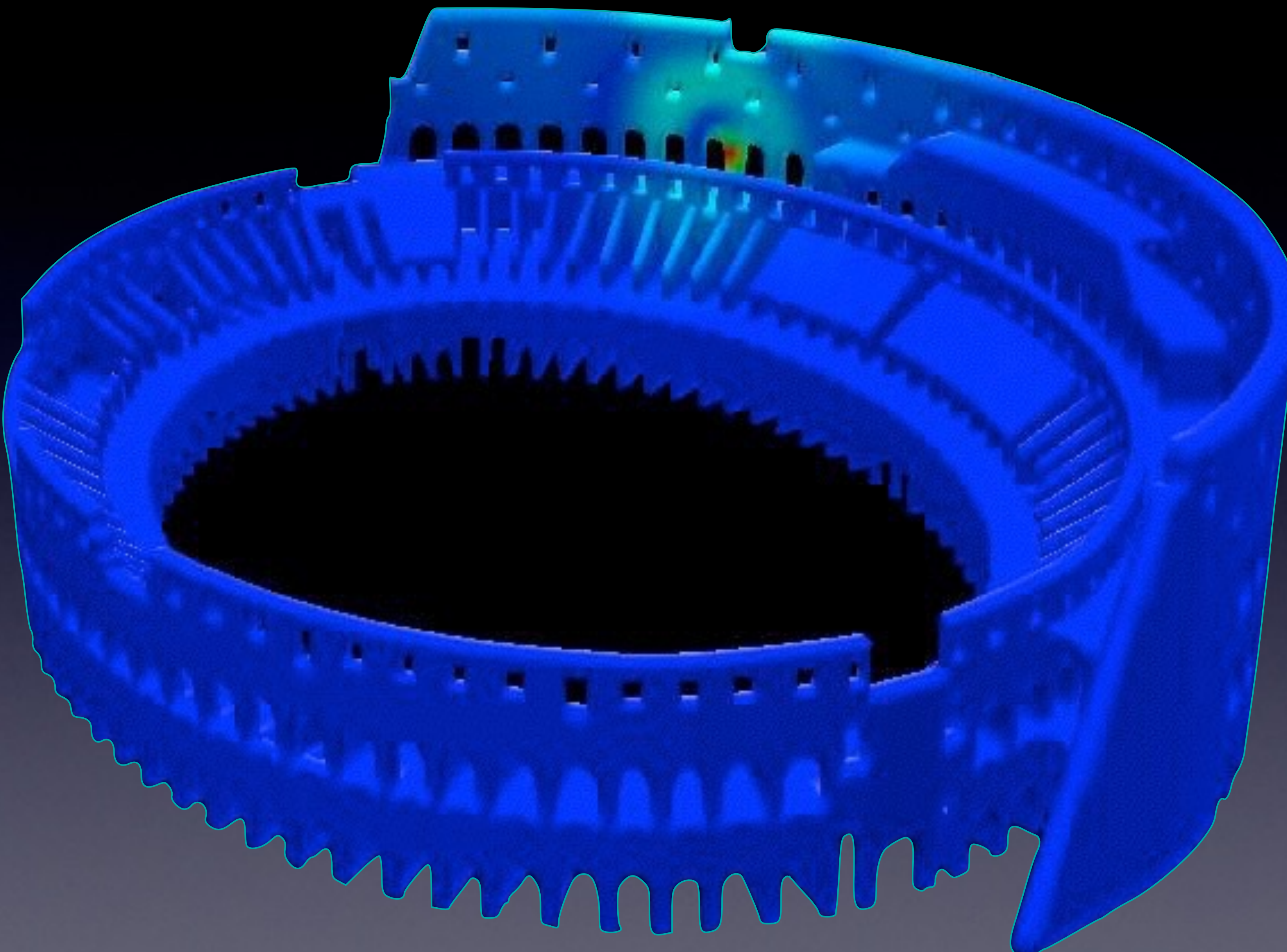
Volcanic eruptions: Vesuvio

Preservation of cultural heritage
Mathematics is non-distructive

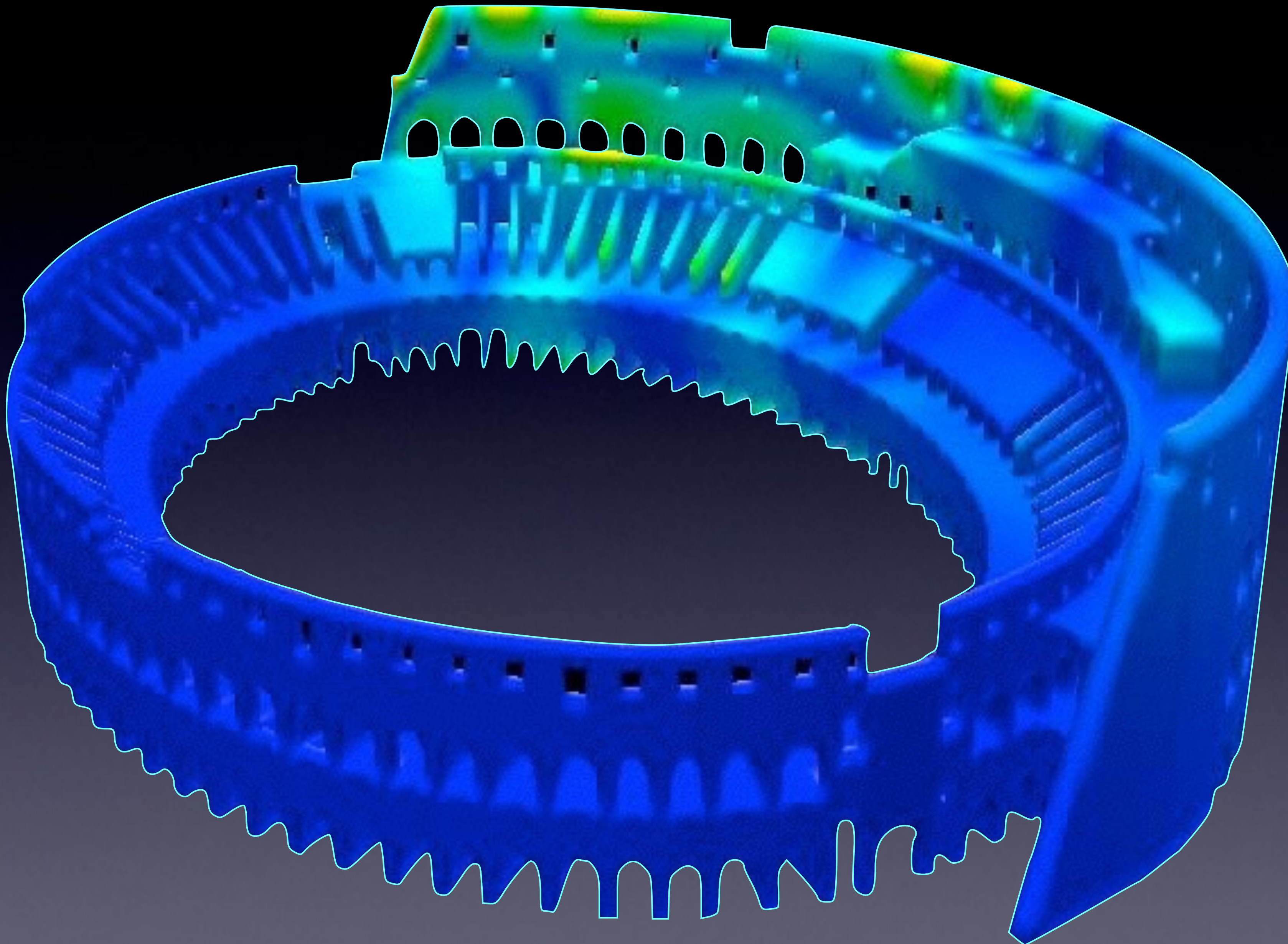




Maggio-Massidda-Fotia @ CRS4

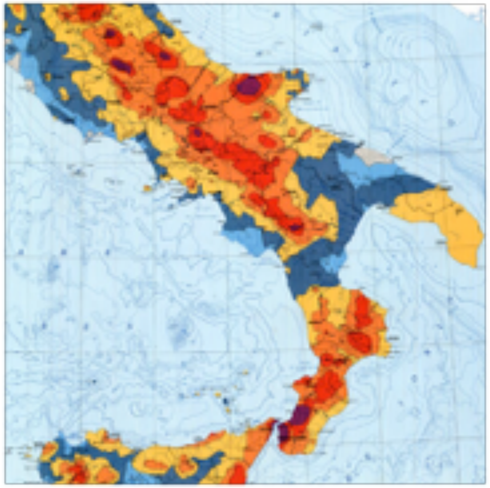


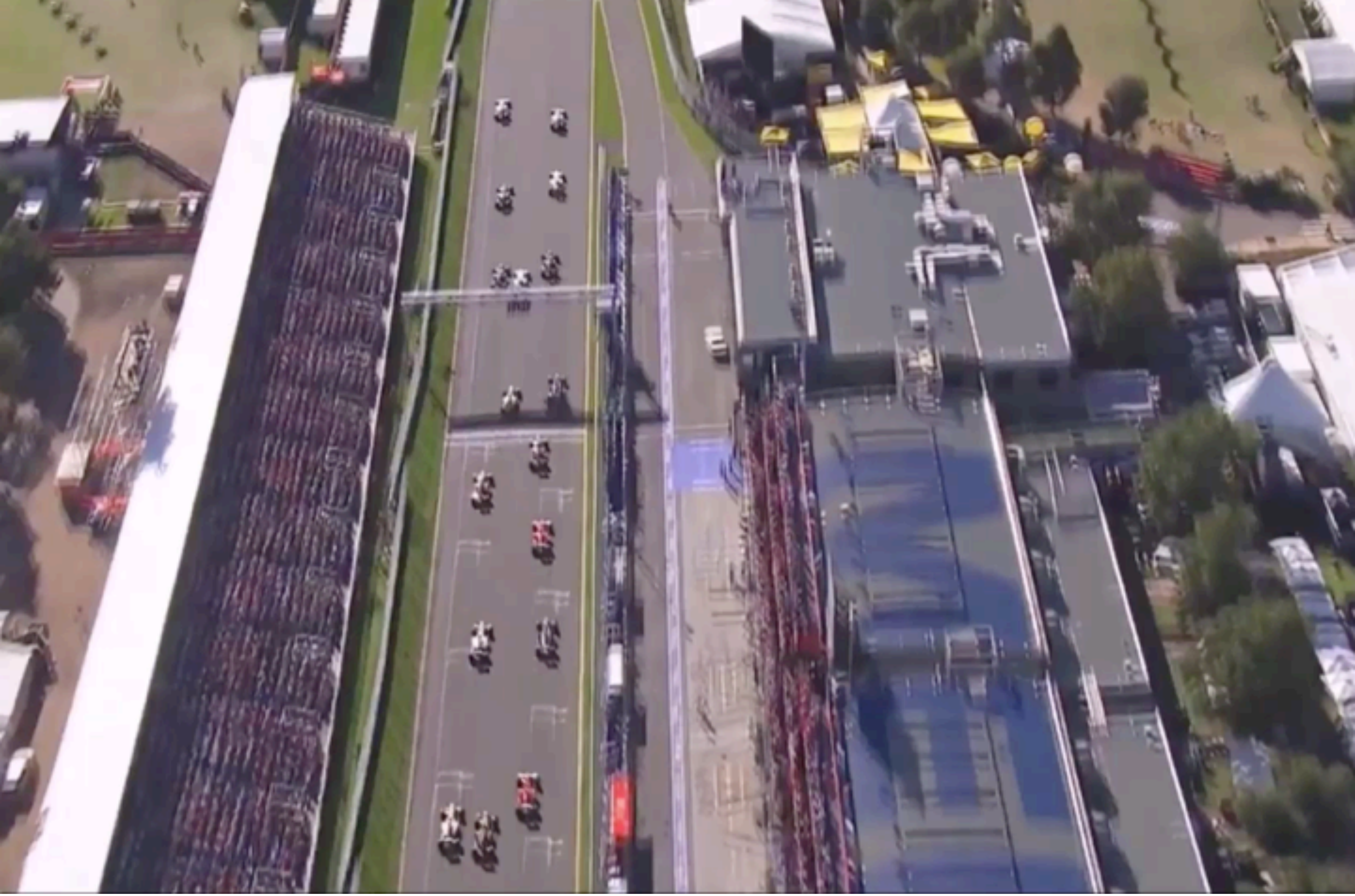
Maggio-Massidda-Fotia @ CRS4

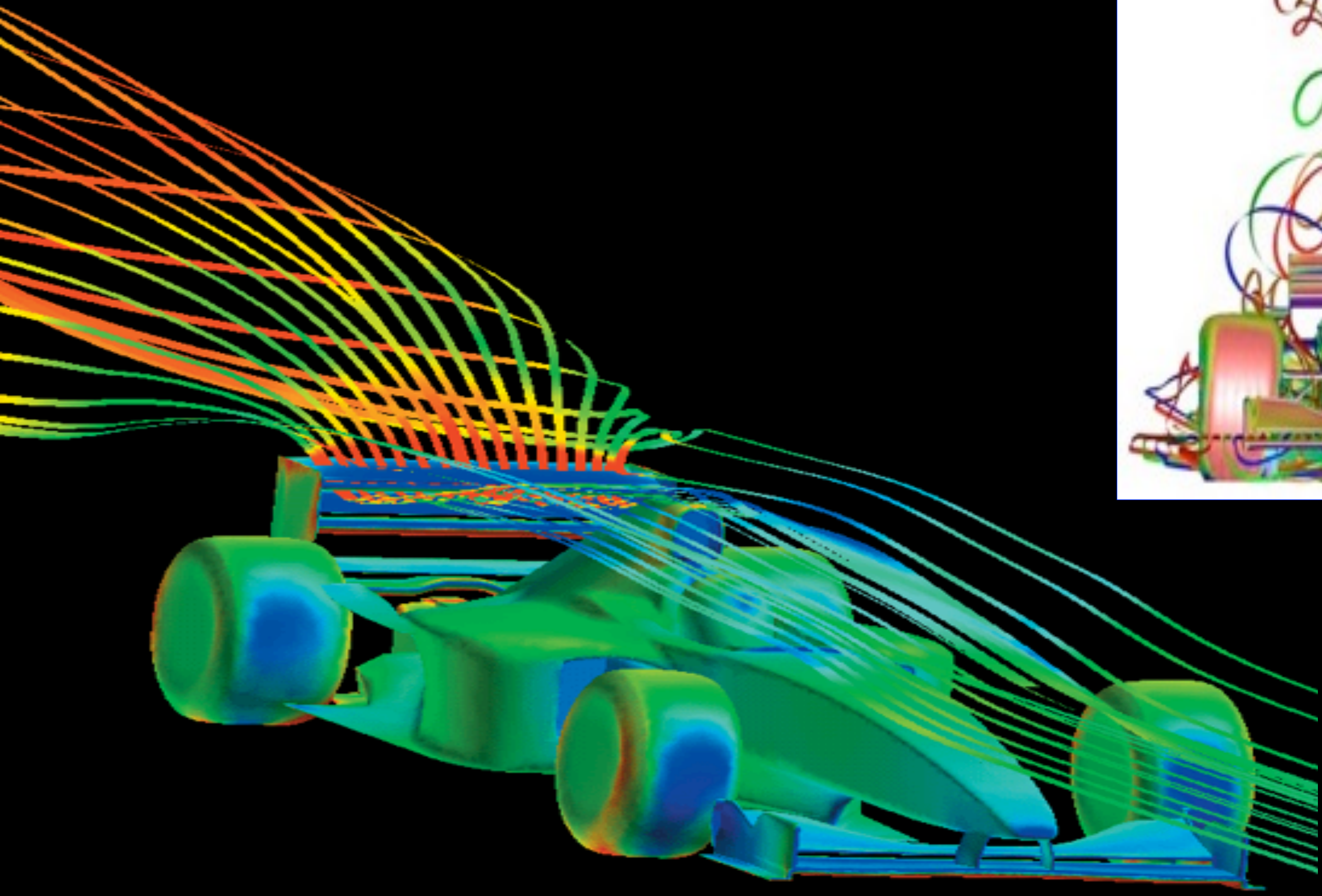
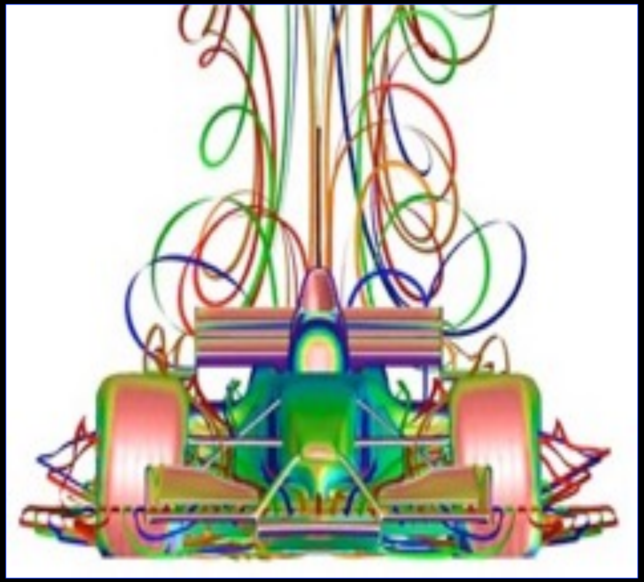


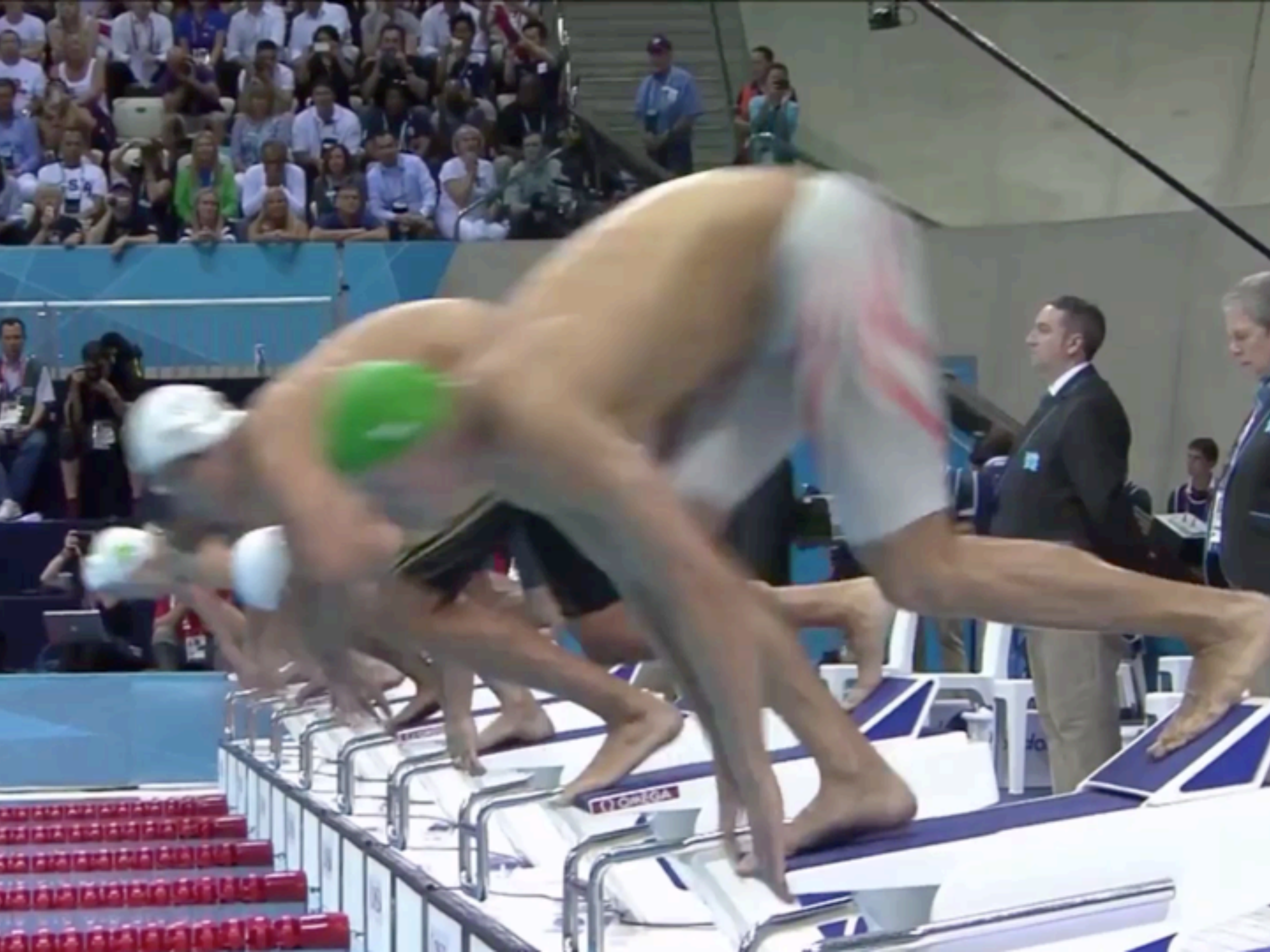
Maggio-Massidda-Fotia @ CRS4

Mathematics
and
sports...

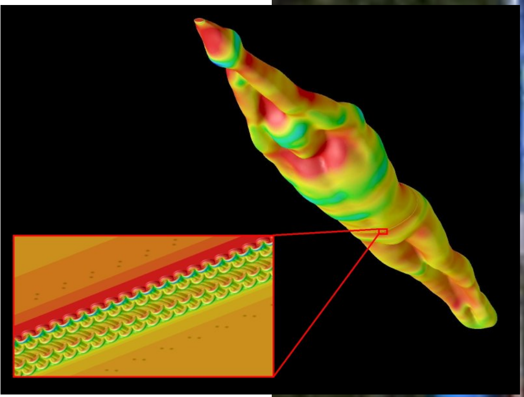





OMEGA

Real swimmers, real swimmers... (N.Parolini, A.Veneziani)



POWERSKIN Xtreme

America's Cup


Alinghi

America’s Cup

alinghi

EPFL
ÉCOLE POLYTECHNIQUE FÉDÉRALE DE LAUSANNE
OFFICIAL SCIENTIFIC ADVISOR
ALINGHI DEFENDER 32nd AMERICA'S CUP


infonet
UBS

Le equazioni dei fluidi

Air Phase

$$\frac{\partial(\rho_a \boldsymbol{u}_a)}{\partial t} + \boldsymbol{\nabla} \cdot (\rho_a \boldsymbol{u}_a \otimes \boldsymbol{u}_a) - \boldsymbol{\nabla} \cdot \boldsymbol{T}_a(\boldsymbol{u}_a, p_a) = \rho_a \boldsymbol{g}$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u}_a = 0$$

Interface Conditions

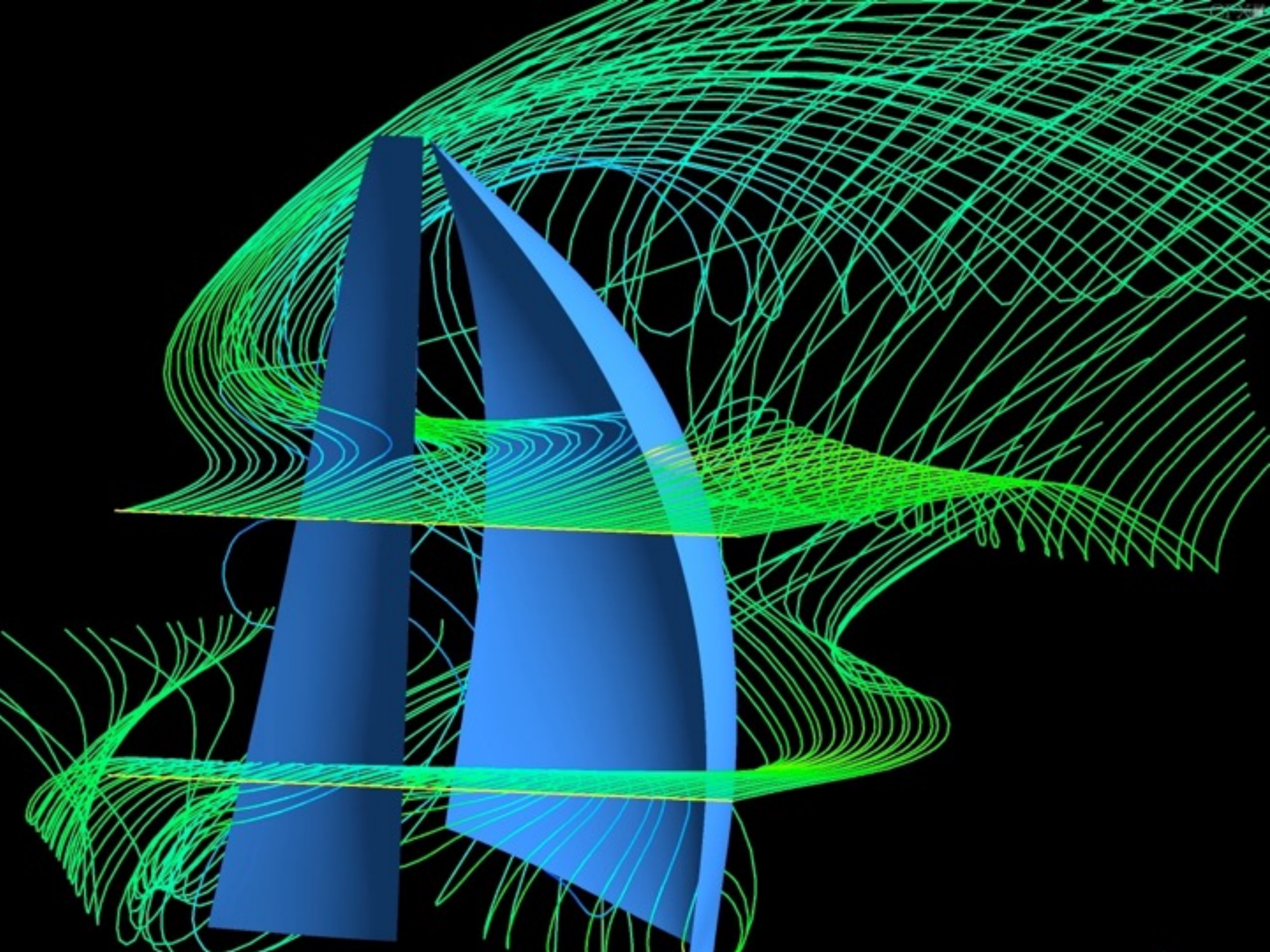
$$\boldsymbol{u}_a = \boldsymbol{u}_w \quad \text{on } \Gamma$$

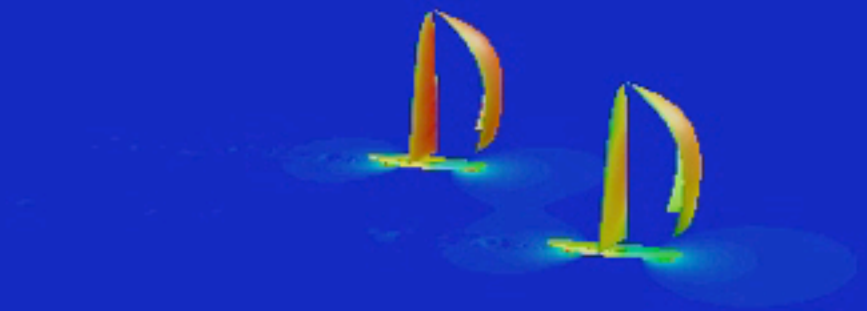
$$\boldsymbol{T}_a(\boldsymbol{u}_a, p_a) \cdot \boldsymbol{n} = \boldsymbol{T}_w(\boldsymbol{u}_w, p_w) \cdot \boldsymbol{n} + \kappa\sigma\boldsymbol{n} \quad \text{on } \Gamma$$

$$\frac{\partial(\rho_w \boldsymbol{u}_w)}{\partial t} + \boldsymbol{\nabla} \cdot (\rho_w \boldsymbol{u}_w \otimes \boldsymbol{u}_w) - \boldsymbol{\nabla} \cdot \boldsymbol{T}_w(\boldsymbol{u}_w, p_w) = \rho_w \boldsymbol{g},$$

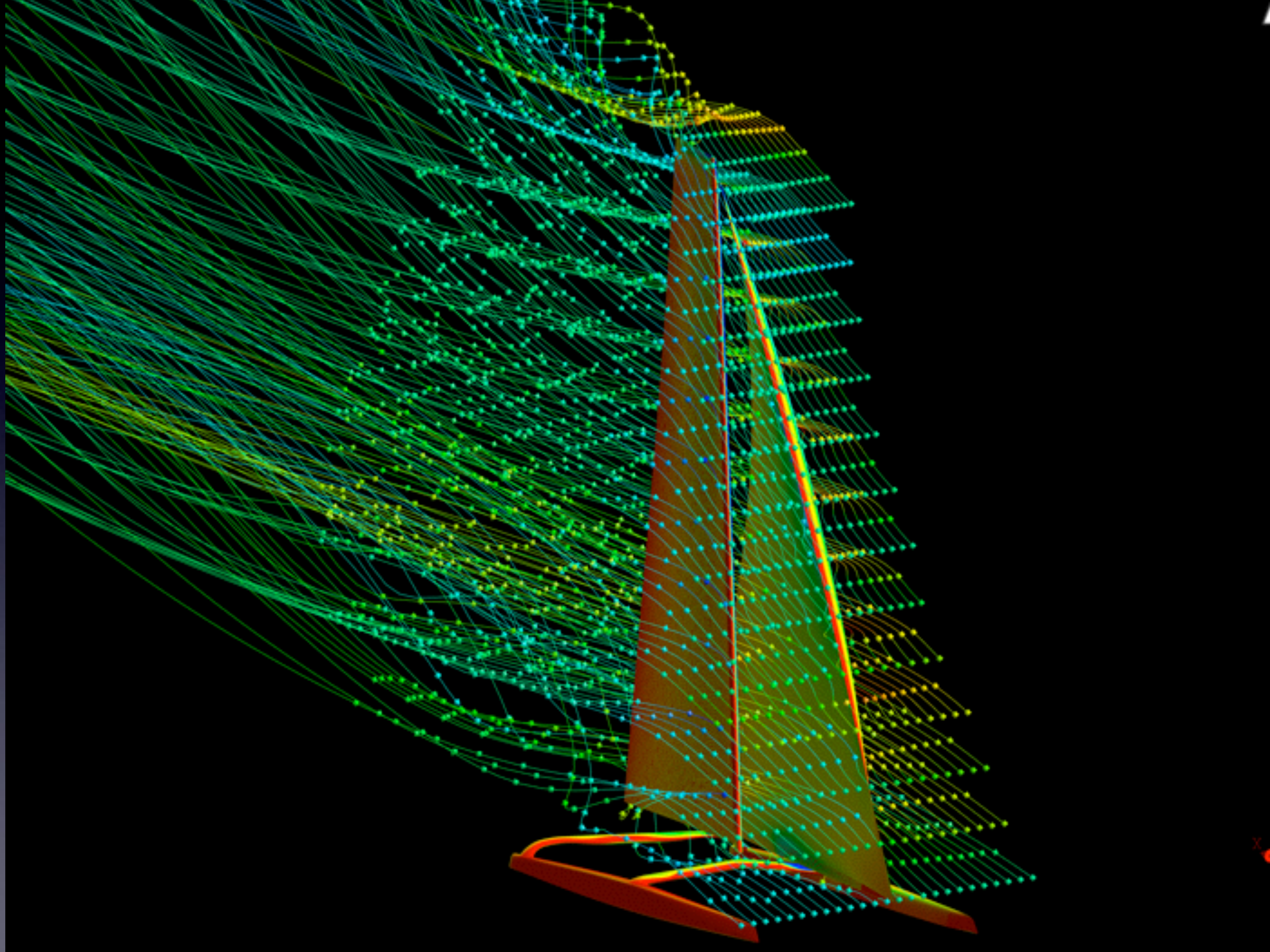
$$\boldsymbol{\nabla} \cdot \boldsymbol{u}_w = 0,$$

Water Phase

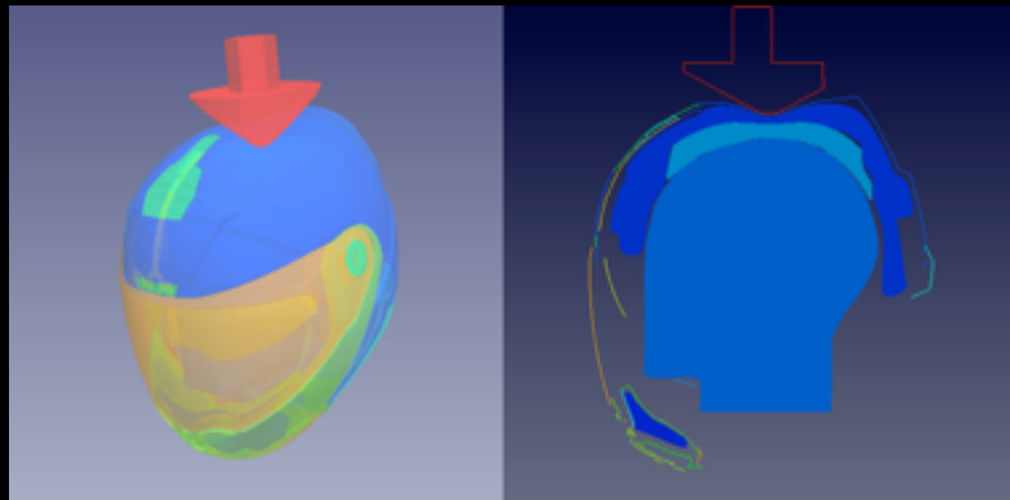


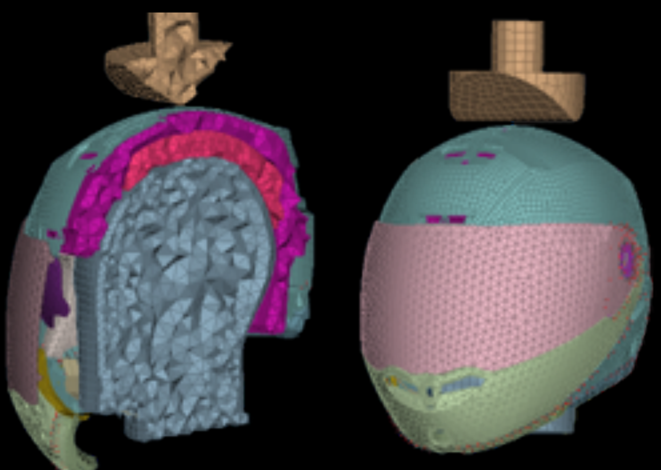


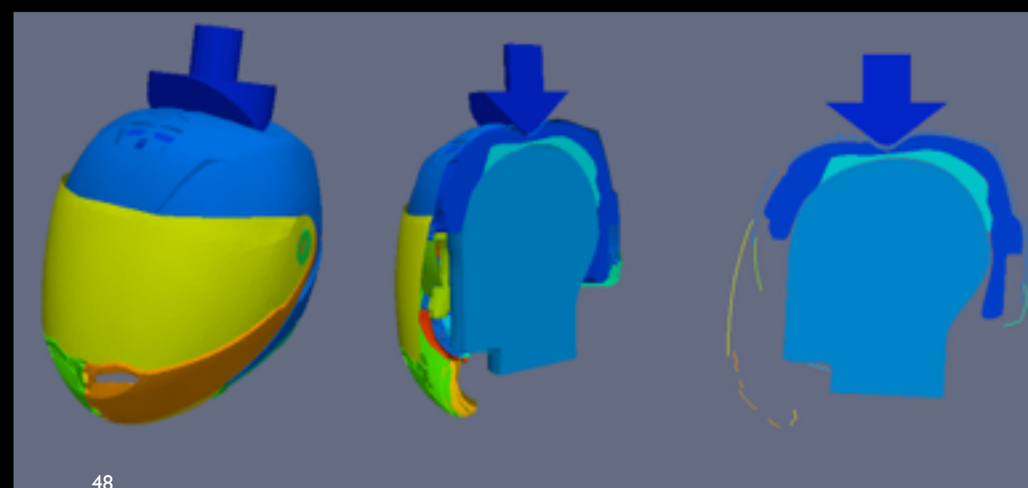

VD 915

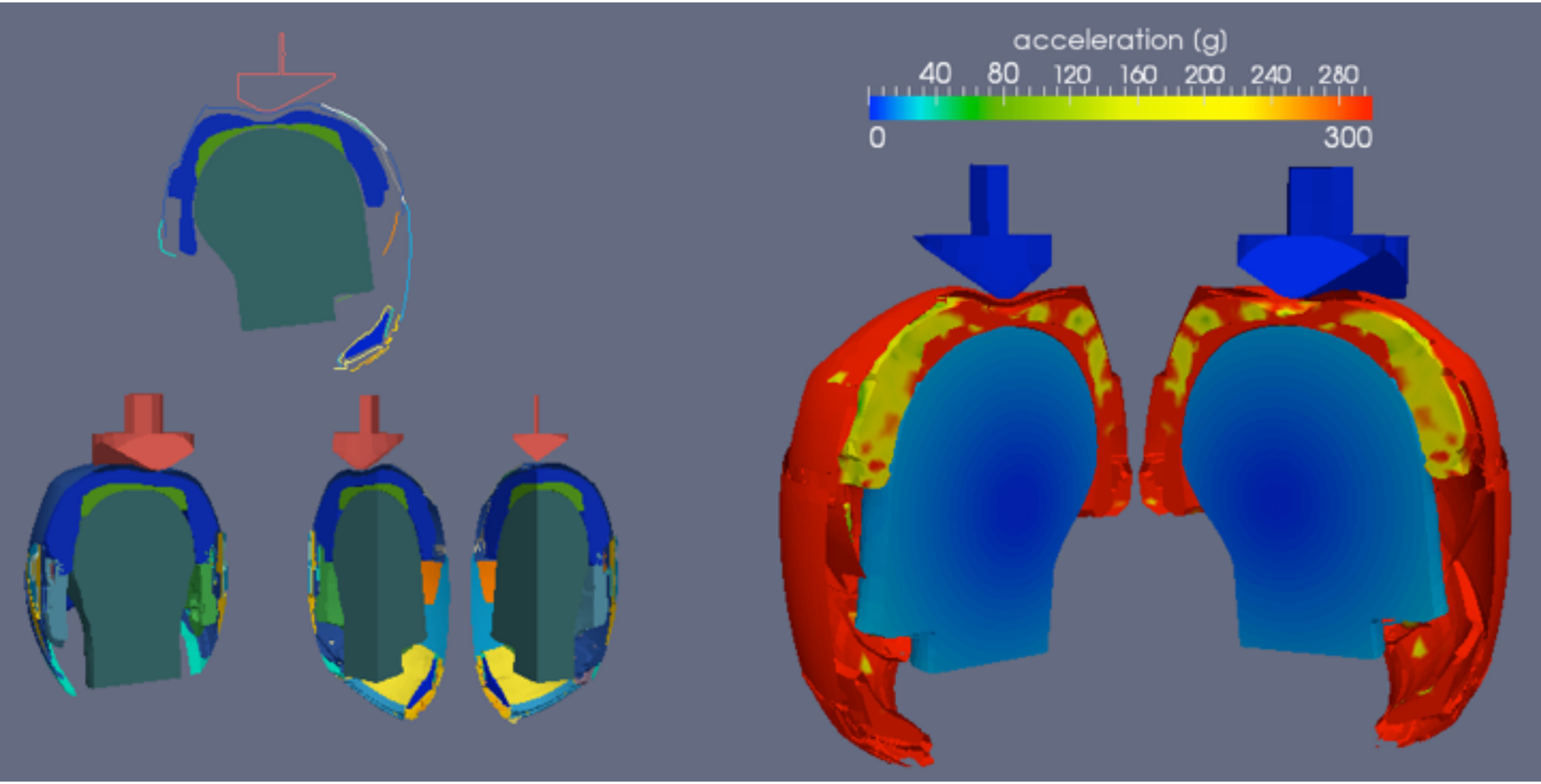


Moto Helmets: crash analysis

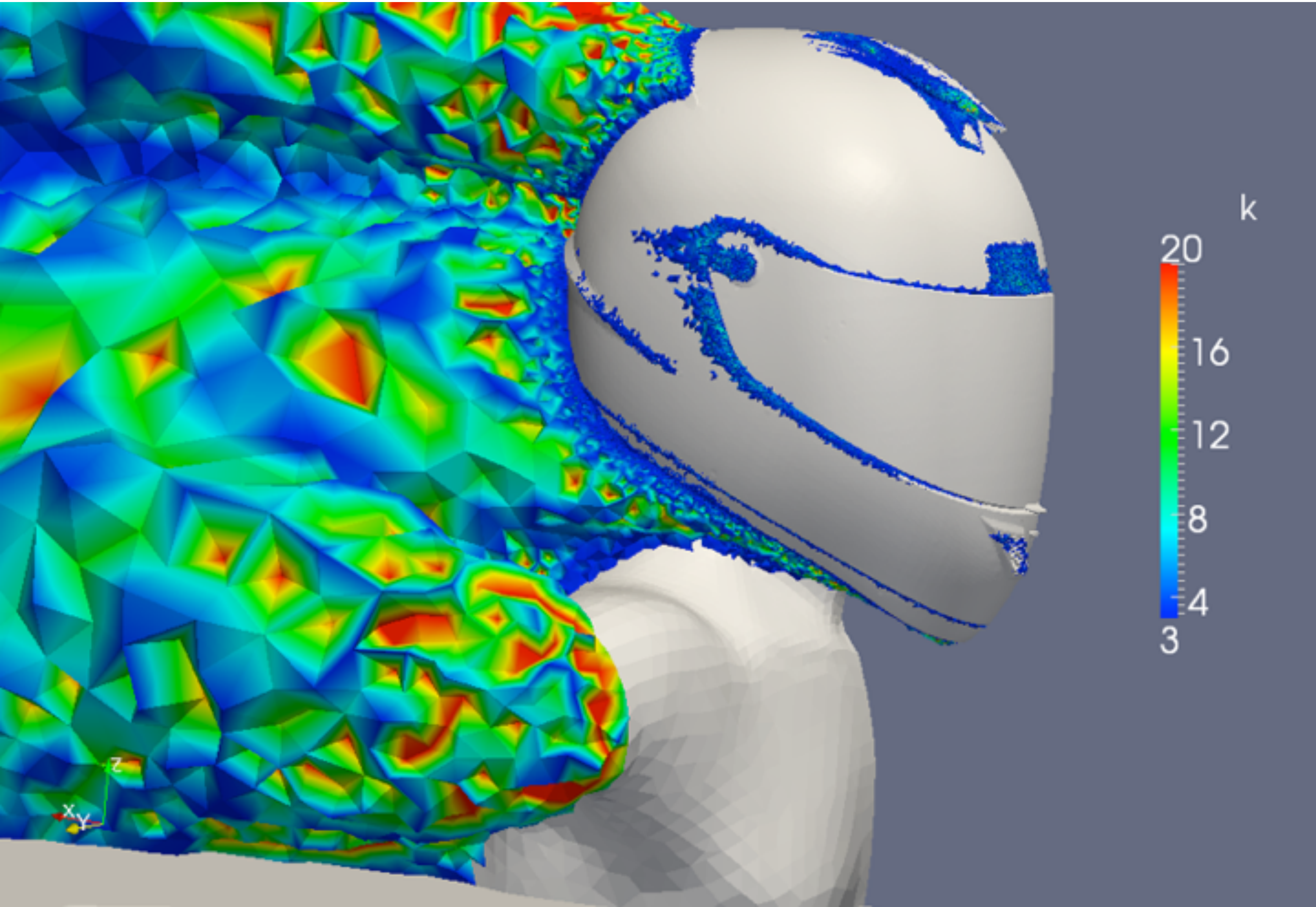




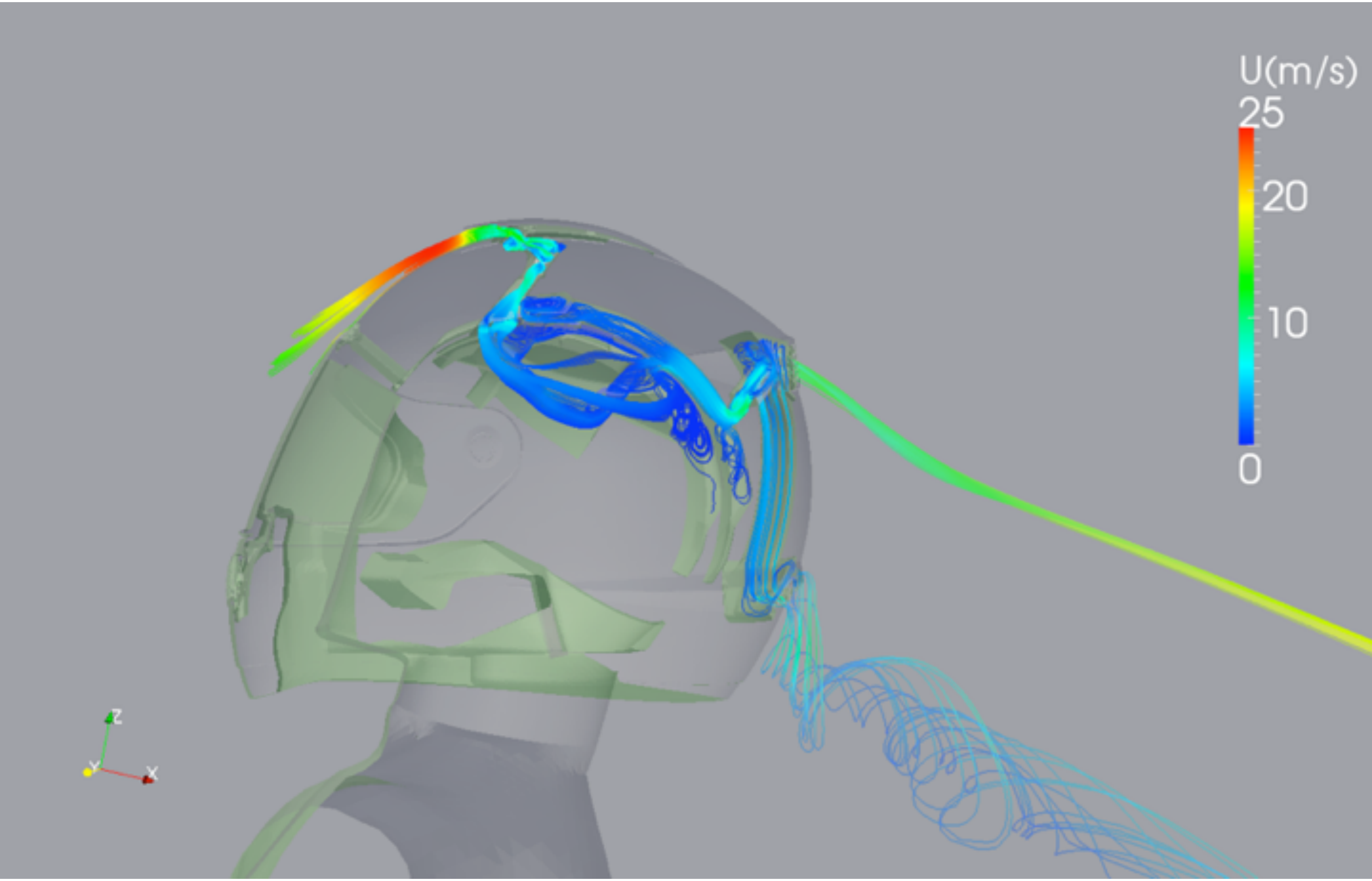



acceleration (g)
40 80 120 160 200 240 280
0
300

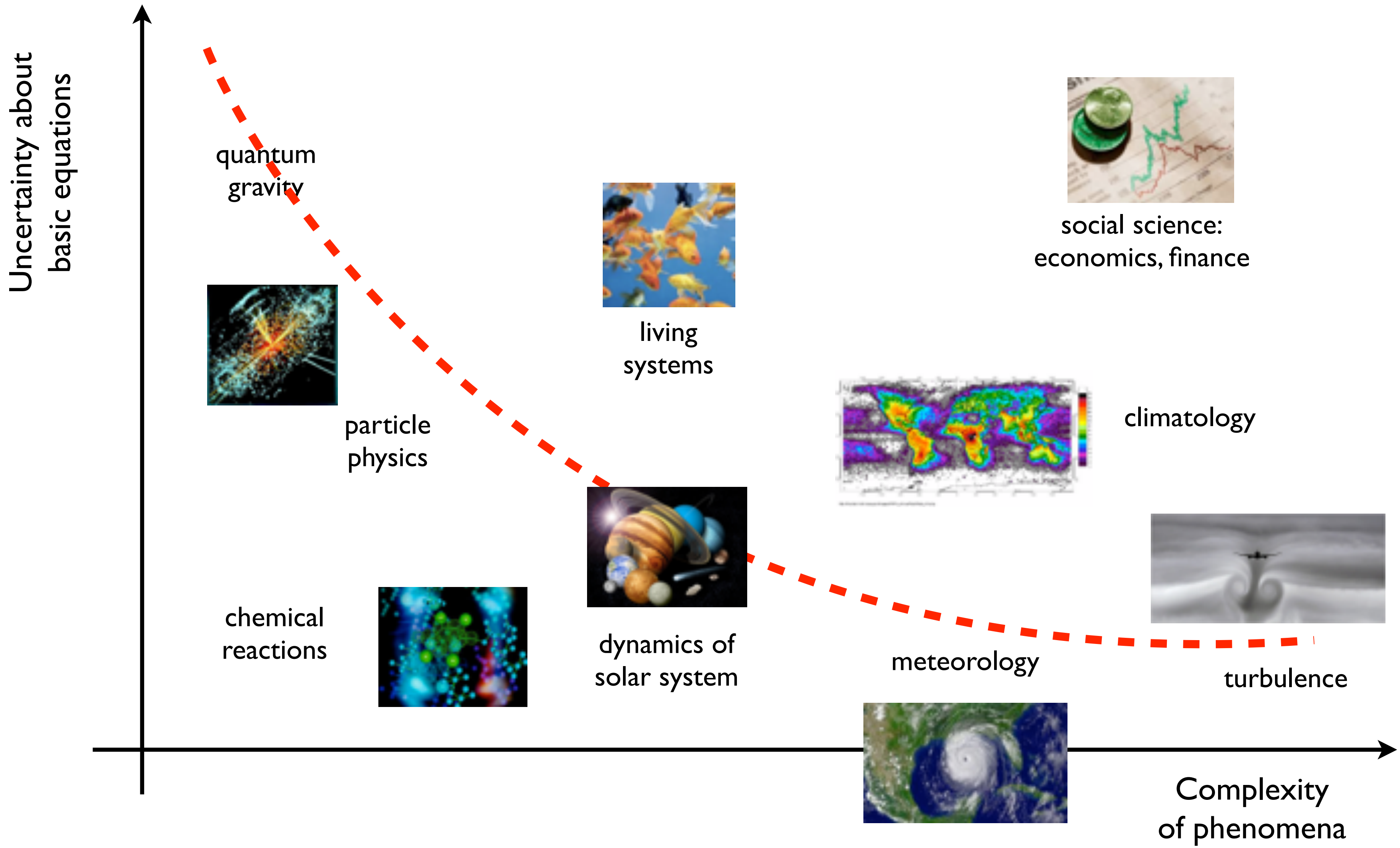
Moto Helmets: aeroacoustics



Moto Helmets: internal ventilation



Where do we stand



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