Simple maths for complex flows

János VAD PhD, DSc
full professor
vad.janos@gpk.bme.hu

Budapest University of Technology and Economics (BME)
Faculty of Mechanical Engineering
Department of Fluid Mechanics

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OUTLINE

- Hardcore (HC) fluid mechanics: an introduction
- Three representative practical case studies, with various working fluids
  - Water mill in the Danube: **water**
  - Clean room: **air**
  - Dryer: mixture of **hot flue gas + air**
- Summary: from a perspective of mathematics, in an evolutionary approcah

- An interactive seminar
- Please feel free to put questions and give comments intermediately (sufficient timeframe)
- Interactions: questions and answers, discussion
- ? ***„Organized interaction”:*** ***+ Interactions***
  26 questions
- → Premium scores for you
- → You will be declared as a titular HC fluids engineer
- → Valuable prizes

Dr. János VAD, vad.janos@gpk.bme.hu

HARDCORE FLUID MECHANICS

? *What does „hardcore” mean ?*

HARDCORE

In music: „HC is generally faster, thicker, and heavier than earlier punk rock.” (*Wikipedia*)

HARDCORE FLUID MECHANICS

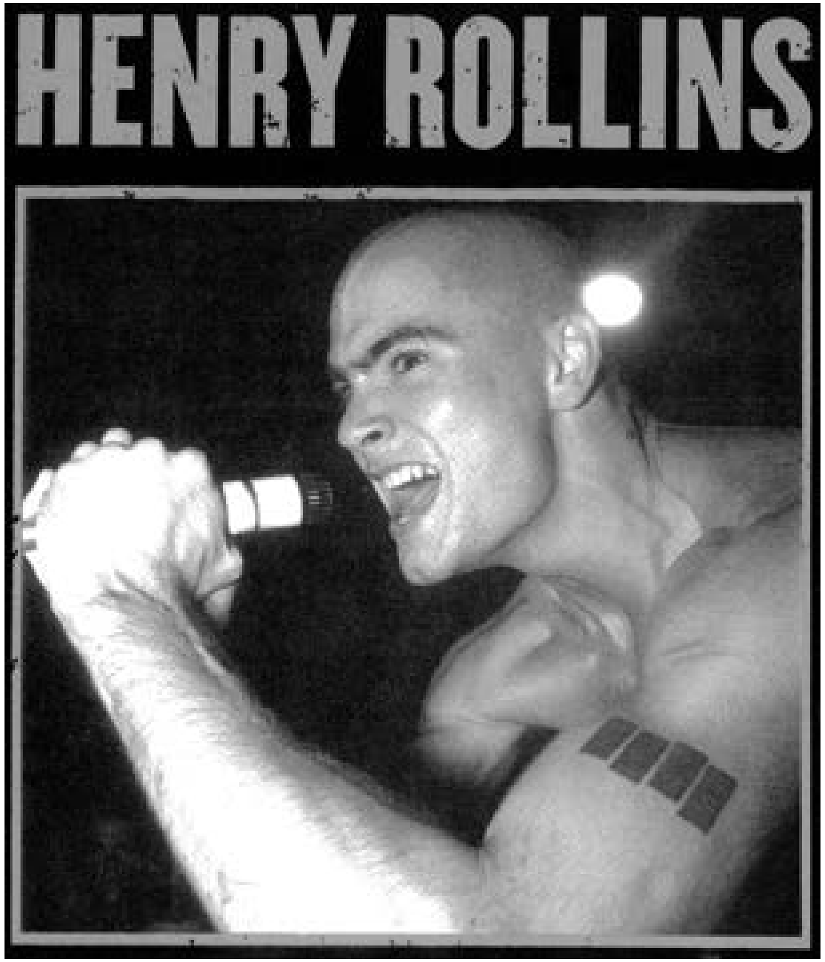
BME Department of Fluid Mechanics:

a „hardcore” group of fluids engineering scene

- •Committed to engineering applications
- •Firm in solving practical problems
- •No hesitation – making decisions, often rapidly

→ **Relevant, brief, quick practical approximations**

- [For refinements of engineering calculations, validated by detailed experiments: advanced physics + mathematics: analysis; Computational Fluid Dynamics (CFD), …]
- Basic physics and maths: primary-school-level → middle-school-level → …
- Minimal modelling: *Einstein:* „A model should be as simple as it can be but no simpler”
- Aims:
  - Quick and brief (order-of-magnitude) estimations (even enabling mental arithmetic)
  - Confirmation ↔ rebuttal of feasibility of operational / constructional concepts
  - Preliminary design of parameters
  - Preliminary design of distributions
  - … and more
- „Keep it simple”: old-school methods and tools
  - Available in the past centuries!
  - Available almost everywhere for you, being at your hand – even in a desert island!
- WHAT TOOLS?

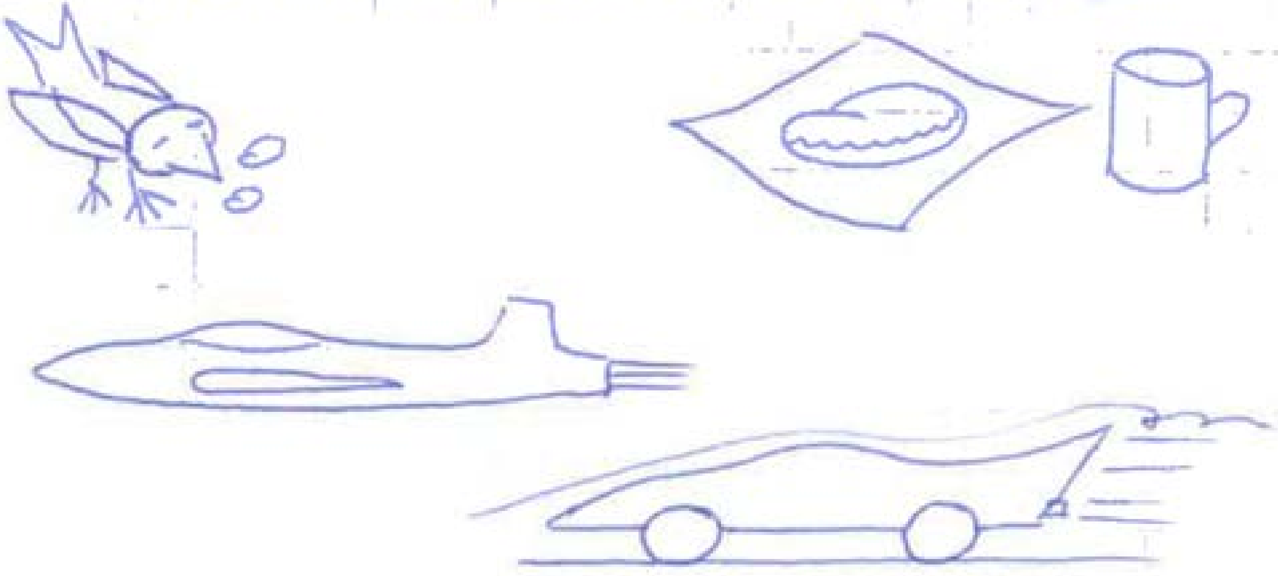


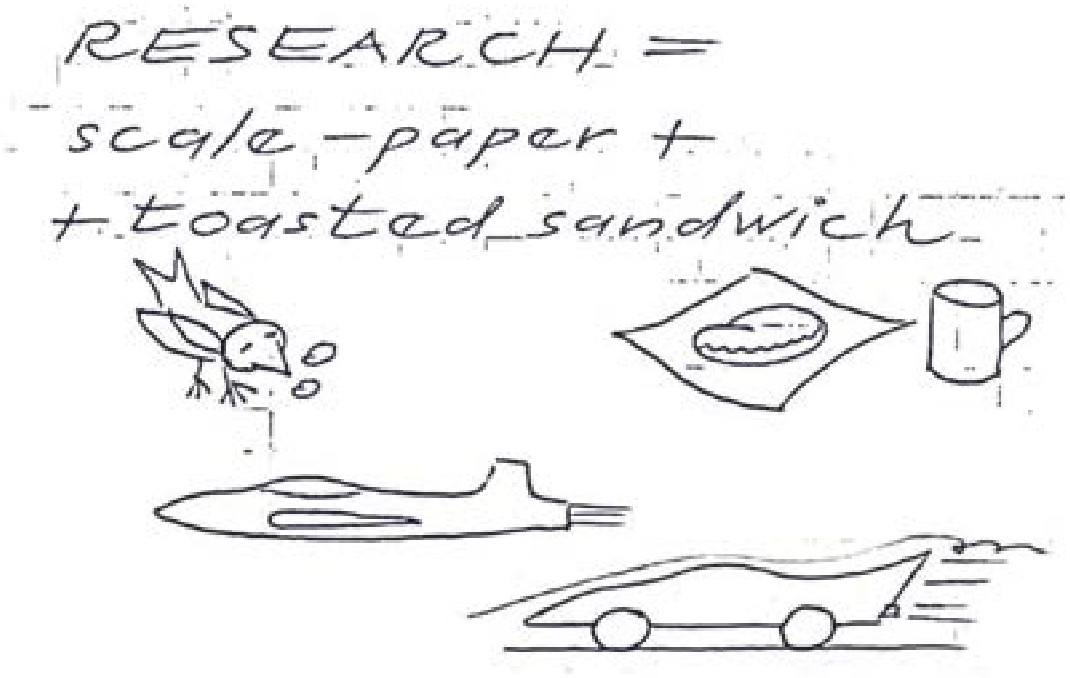
„Keep
•your blood clean,
•your body lean,
•and your mind sharp.”

•Use your brain
•Be brave (to make sketches etc.)
•Be critical / self-critical

? ***Break – ?***

RESEARCH =
scale-paper +
+ toasted sandwich – *Dr. Gergely KRISTÓF*





+ a pen / pencil

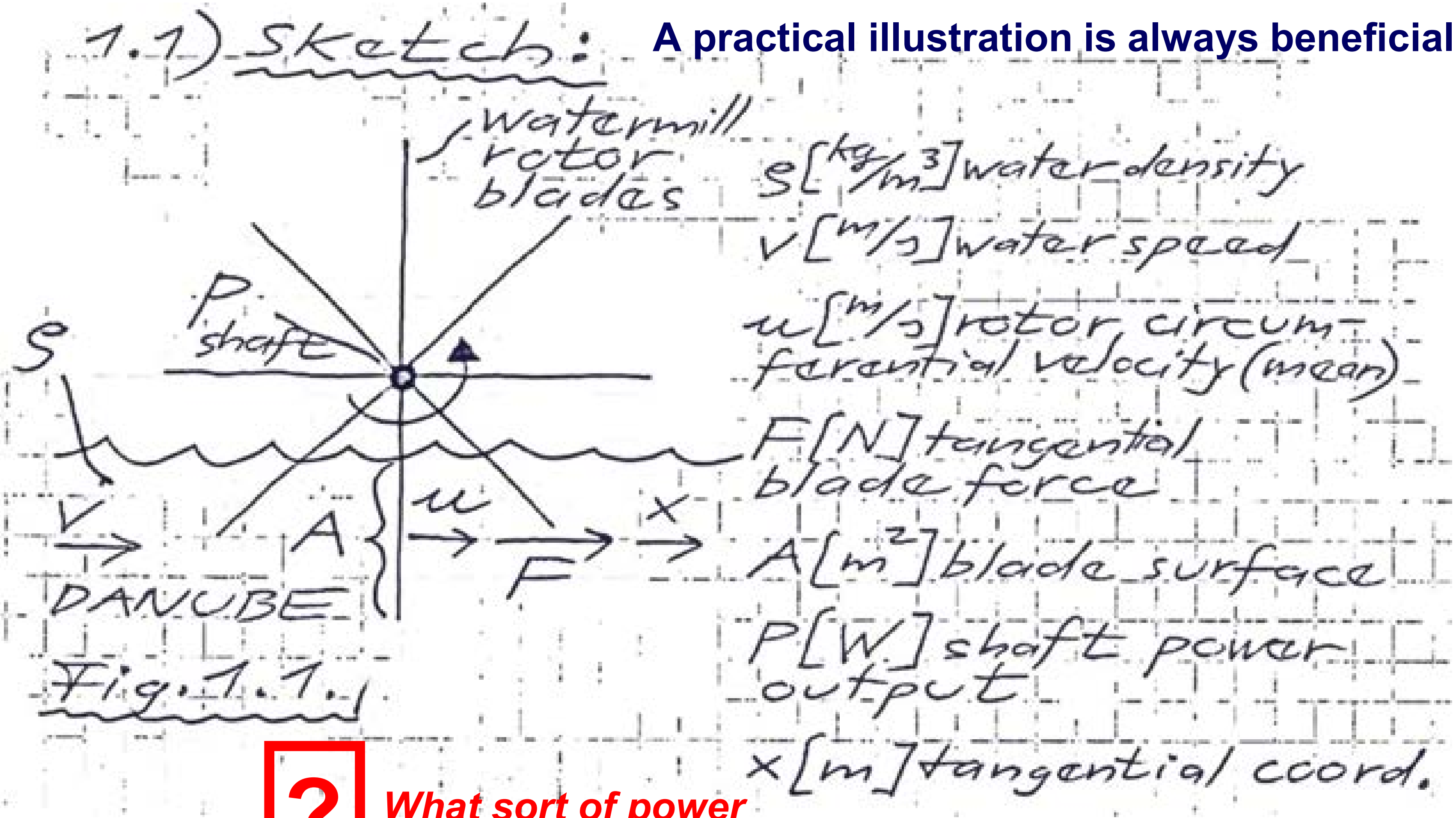
- Three case studies
  - Engineering relevance, AND
  - Strengthening your social relations by means of FLUID MECHANICS:
    - How making new friends / a girlfriend / a boyfriend?
    - How promoting your mobile phone communication?
    - How contributing to the morning cornflakes of your family?
- FLUID MECHANICS: no „mistification”, no „rocket science” (at least in these examples): real life, „everyday miracles”
- FLUID MECHANICS: sexy, cool, trendy
- An intellectual pleasure, an exciting game – ENJOY!

II. CASE STUDY 1: WATER MILL IN THE DANUBE

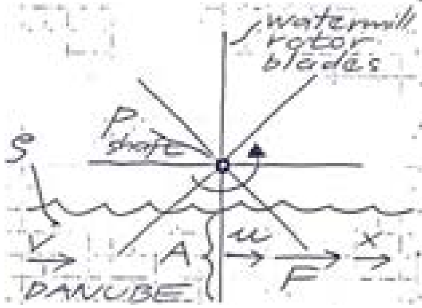
- The story started at a party on a New Years' Day: eating, drinking, making music…
- Artists: painters, sculptors, photographers, a glass blower…
- + A single mechanical engineer (in fluid mechanics)
- Artistic inspirations: flying birds, waving water, blazing flames, billowing clouds…
- The idea by the glass blower:
  - A water mill on a boat in the Danube
  - Generating electricity for an electric glass smelter furnace
  - Glass blowing as tourist attraction + onboard restaurant for tourists…

- Is the idea technically feasible?
- Only some minutes are available for the answer → then losing the artists' interest
- Tools: a piece of paper napkin, + a pen
- Very brief outline of the following description on the paper napkin

A practical illustration is always beneficial



What sort of power is the final aim ?



1.2) Formulation of problem:

P prescribed (demanded) ←

a) How to set "u" for maximum obtainable power P_{max}? ← operational demand

b) A = ? ← geometrical (design) demand

? Examples for loss sources ?

1.3) Assumptions:

"Idealistic" scenario: all losses neglected; maximum utilization of blades (no blade interference etc.)

$$P = P_{id} \leftarrow u;\ \underbrace{A_{min}}_{\text{minimum necessary}} \qquad (1.1)$$

? F; u → P ?

1.4) Solution / description:

1.4) Solution/description:

$$P = F \cdot u \; [W] = \left[\frac{J}{s}\right] = \left[\frac{N \cdot m}{s}\right] \qquad (1.2)$$

$F = ?$ ← Newton II.

? *F ~ temporal rate of change of what ?*

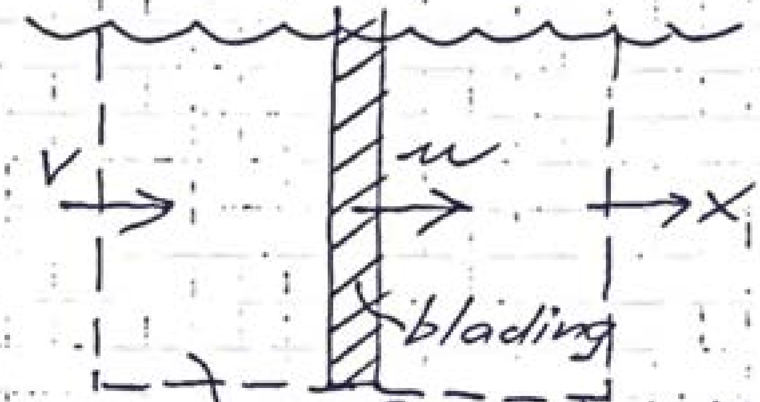


Fig. 1.2.

? *Mass; velocity → momentum ?*

Δm [kg]: inlet water mass in duration of Δt [s]

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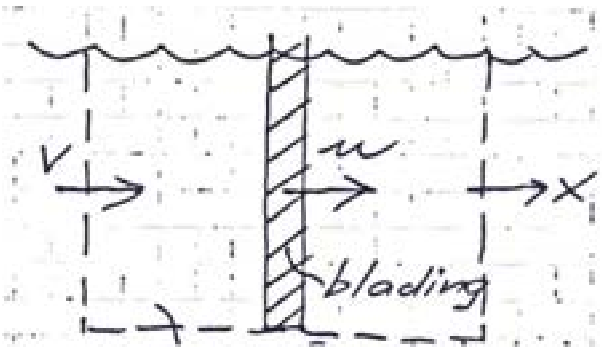

x-wise (tangential) inlet momentum:

$$\Delta I_{in} = \Delta m \cdot v \qquad (1.3)$$

$$[kg] \cdot [m/s]$$

Inlet x-wise momentum flux:

$$\frac{\Delta I_{in}}{\Delta t} \hat{=} \dot{I}_{in} = \frac{\Delta m}{\Delta t} \cdot v \qquad (1.4)$$

$$[kg/s] \cdot [m/s] = [kg \cdot \frac{m}{s^2}]$$

Where

$$\frac{\Delta m}{\Delta t} \hat{=} q_m \text{ mass flow rate } [\frac{kg}{s}] \qquad (1.5)$$

$$q_m [kg/s] = \rho \cdot v \cdot A \qquad (1.6)$$

$$[\frac{kg}{m^3}] \cdot \underbrace{[\frac{m}{s}] \cdot [m^2]}_{v \cdot A \,\hat{=}\, q_V \text{ volume flow rate}} = [\frac{kg}{s}] \qquad (1.7)$$

$$[\frac{m}{s}] \cdot [m^2] = [\frac{m^3}{s}]$$

?

ρ for water ?

(1.4):

momentum flux = mass flow rate × velocity (1.8)

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$\dot{I}_{in} = q_m \cdot v = (\rho \cdot v \cdot A) \cdot v \quad (1.9)$

Outlet momentum flux:

$\dot{I}_{out} = q_m \cdot \underbrace{u}_{\text{constrained by the blading}} = (\rho \cdot v \cdot A) \cdot u \quad (1.10)$

Newton II: $[N = \frac{kg \cdot m}{s^2}] = [\frac{kg}{s}] \cdot [\frac{m}{s}]$

$\dot{I}_{in} - \dot{I}_{out} = F = (\rho \cdot v \cdot A) \cdot \underbrace{(v - u)}_{\text{deceleration}} \quad (1.11)$

$(1.2)(1.11) \rightarrow$

$P = (\rho \cdot v \cdot A) \cdot (v - u) \cdot u \quad (1.12)$

a) How to set "u" for maximum P?

- If $u = 0$: $F = F_{max}$ but $P = F \cdot u = 0$ (1.13a)
- If $u = v$: (free running):
  $u = u_{max}$ but $F = 0 \rightarrow P = 0$ (1.13b)

$u = ?$ - A quick, intuitive choice "in between"

? ***What is your hint for u ?***

? ***How to determine u theoretically ?***

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$$u := \frac{v}{2} \qquad (1.14)$$

Justification: (1.12) $\rightarrow$

$$\frac{\partial P}{\partial u} = (\rho \cdot v \cdot A)\cdot \underbrace{\frac{\partial}{\partial u}\left[(v-u)\cdot u\right]}_{(-1)\cdot u + (v-u) = 0} := 0 \qquad (1.15a)$$

$$v - 2u = 0$$

$$\underline{v = 2u} \ \checkmark$$

$$\frac{\partial^2 P}{\partial u^2} : \frac{\partial}{\partial u}(v - 2u) = \underline{-2 < 0} \ \checkmark \qquad (1.15b)$$

maximum! $\qquad (1.15c)$

b) $A_{min} = ?$

(1.12)(1.14) $\rightarrow$

$$P_{maxid} = (\rho \cdot v \cdot A_{min})\cdot\left(v - \frac{v}{2}\right)\cdot\frac{v}{2} =$$

$$= \rho \cdot v^3 \cdot A_{min} \cdot \frac{1}{4} \qquad (1.16a)$$

$$\underline{A_{min} = \frac{4 \cdot P_{maxid}}{\rho \cdot v^3}} \qquad (1.16b)$$

? ***v for the Danube ? – Estimate: let's walk***

? ***Area for the classroom ? – Estimate***

$$A_{min} = \frac{4 \cdot P_{max\,id}}{\rho \cdot v^3} \qquad (1\text{-}16b)$$

1.5) Calculation example:

choice of input data:

$\rho = 10^3\ kg/m^3$

$v = 1\ m/s$

$P_{max\,id}$ = ? Household electric oven: 5 kW → small glass smelter furnace: ≈20 kW = $2 \cdot 10^4$ W

$$A_{min} = \frac{4 \cdot 2 \cdot 10^4}{10^3 \cdot 1^3} = 80\ m^2$$

- Unfeasibly large surface (even for idealistic circumstances) → irrealistic to realize (technical + legislation issues…)
- Issue with *v* on the cube in the denominator! ← Water turbine technology: for rushing rivers; for advanced water turbines with a dam
- Impact on girls and boys at the party
- Hardcore fluid mechanics: an effective tool for making new friends / a girlfriend / a boyfriend

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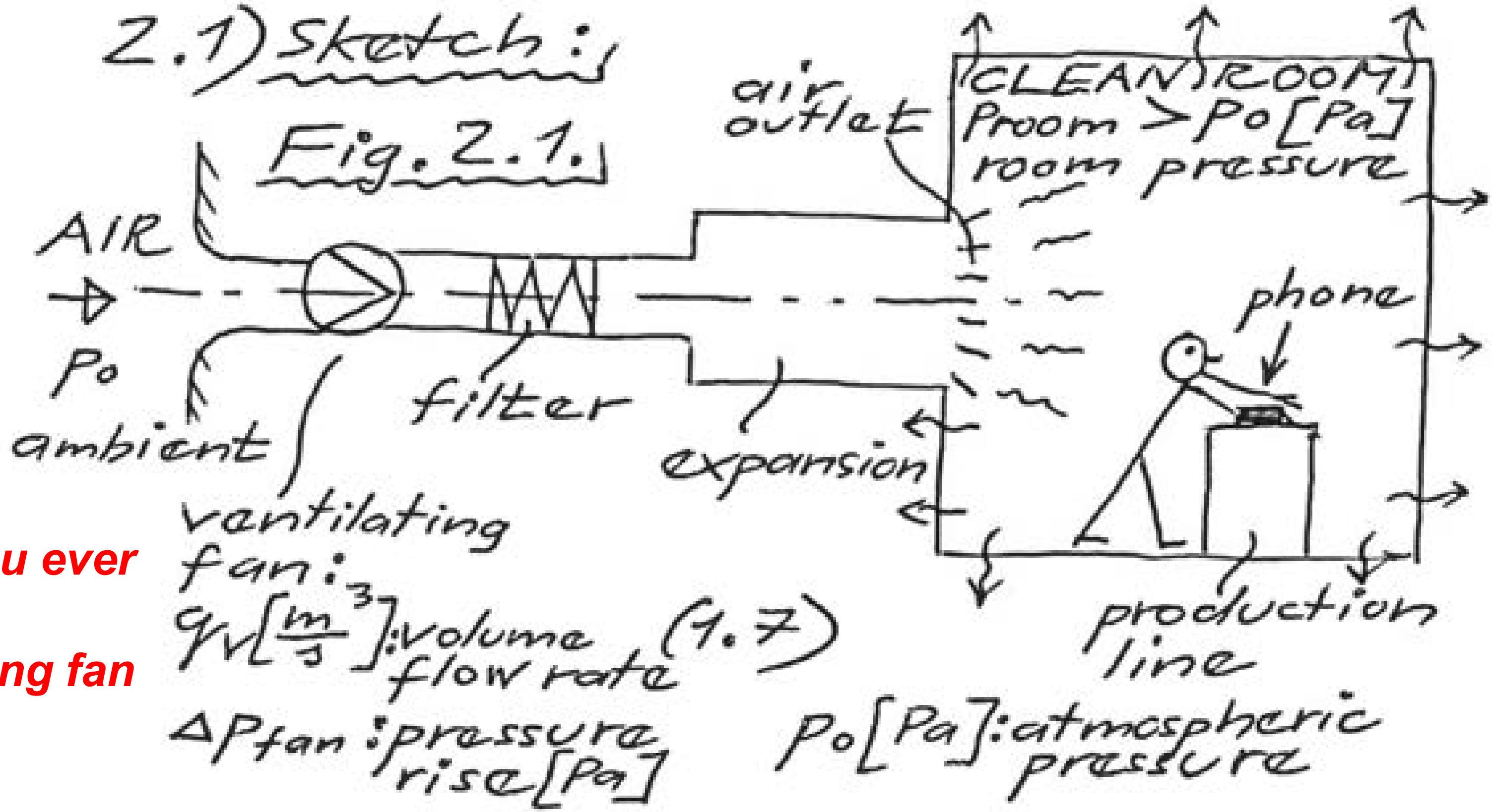

?

Have you ever seen a ventilating fan ?

A practical illustration is always beneficial

?

p_0 estimate ?

Δp_{fan} pressure rise:

- Covering flow friction losses
- Maintaining room overpressure ($p_{room} - p_0$)
  - ~ against the infiltration of contaminated (dusty) ambient air:
  - ~ only outward flow is allowed, via air gaps etc. (no inward flow of unfiltered air!)

2.2) Assumptions,

"Small" changes in pressure and temperature → ρ = constant air density $[kg/m^3]$: incompressible approach

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2.3) Sudden duct expansion

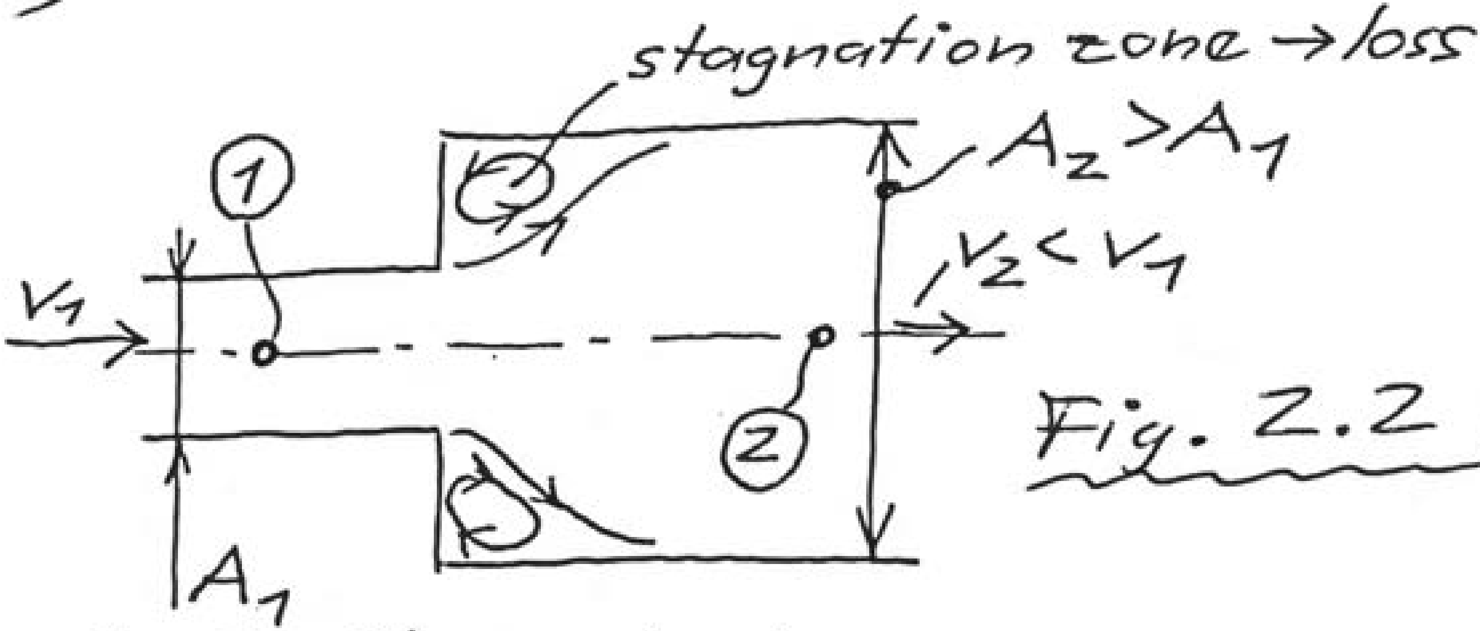

Fig. 2.2

v [m/s] ~ flow velocity
A [m²] ~ cross-sectional area

? ***Why sudden ? What can we spare ? What is cheaper ?***

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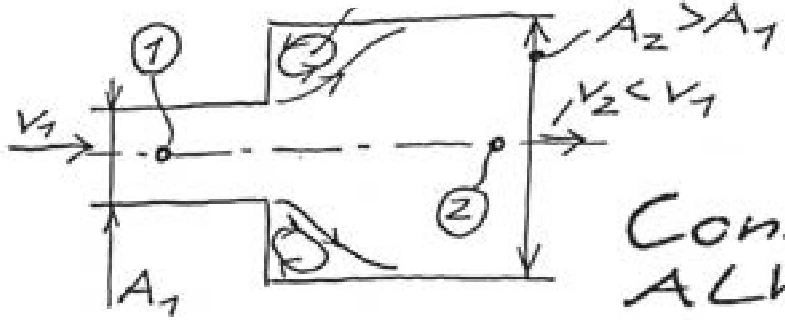

Conservation of fluid mass:
ALWAYS valid in classic mechanics:
Mass flow rate: (1.6)

$$q_{m1} = q_{m2} \; [kg/s] \qquad (2.1a)$$

$$\rho v_1 A_1 = \rho v_2 A_2 \qquad (2.1b)$$

$$v_1 A_1 = v_2 A_2 \qquad (2.1c)$$

$$q_{v1} = q_{v2} \; [m^3/s] \qquad (2.1d)$$

Volume flow rate (1.7)

v_1, A_1: tailored by the fan (given)

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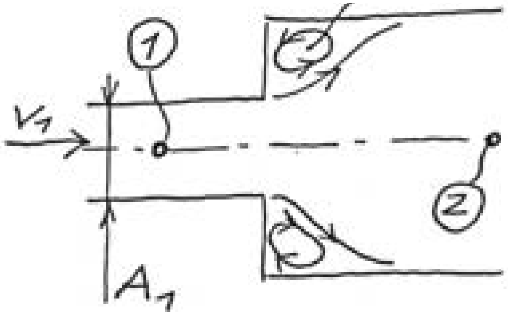


Conservation of energy:

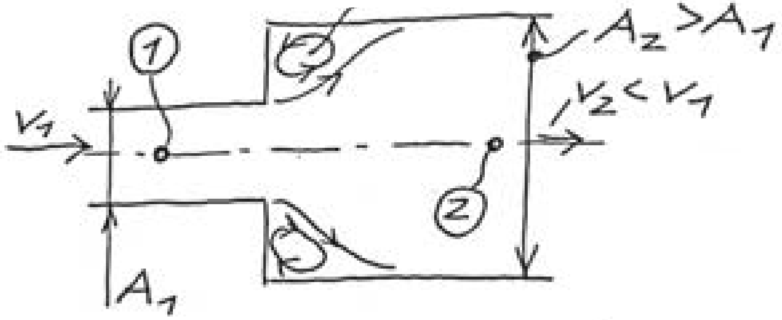
Bernoulli equation 1 → 2 :

$$\underbrace{p_1}_{\text{static pressure}} + \underbrace{\rho \frac{v_1^2}{2}}_{\substack{\text{dynamic pressure} \\ (\leftarrow \text{kinetic energy})}} = p_2 + \rho \frac{v_2^2}{2} + \underbrace{\Delta p'}_{\substack{\text{pressure loss} \\ (\leftarrow \text{flow friction})}} \qquad (2.2a)$$

$$\left[\frac{v^2}{2}\right] = \frac{J}{kg}$$

$$\left[\rho \frac{v^2}{2}\right] = \frac{J}{kg} \cdot \frac{kg}{m^3} = \frac{J}{m^3}$$

$$\underbrace{p_2 - p_1}_{\text{pressure recovery}} \stackrel{\wedge}{=} \Delta p = \rho\left(\frac{v_1^2}{2} - \frac{v_2^2}{2}\right) - \Delta p' \qquad (2.2b)$$



?

An example: lower velocity → lower loss → less cost ?

Benefits of expansion:

- $v_2 < v_1$ slower outflow; no draft → good for human health
- less velocity: $v_2 < v_1$ → moderate flow friction loss (energetically more favourable)
- $v_2 < v_1$: $p_2 > p_1$: pressure recovery Δp : contributes to $p_{room} > p_0$

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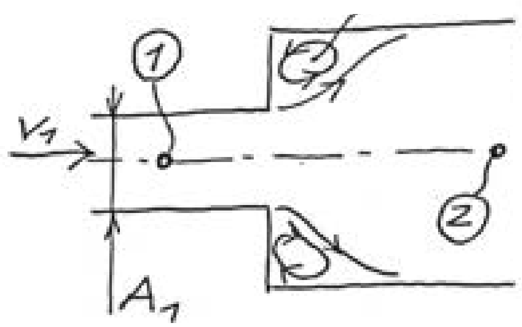

Pressure recovery coefficient:

$$C_p = \frac{p_2 - p_1}{\rho \frac{v_1^2}{2}} = \frac{\Delta p}{\underbrace{\rho \frac{v_1^2}{2}}_{\text{inlet specific kinetic energy}}} \qquad (2.3)$$

"Pressure increasing capability per unit inlet kinetic energy"

Area ratio: $n_{AR} = \frac{A_2}{A_1}$ (2.4)

(2.1c) →

$$n_{AR} = \frac{v_1}{v_2} \qquad (2.4a)$$

2.4.) Formulation of problem

How to select n_{AR} to obtain maximum C_p? "Energetically efficient behaviour"

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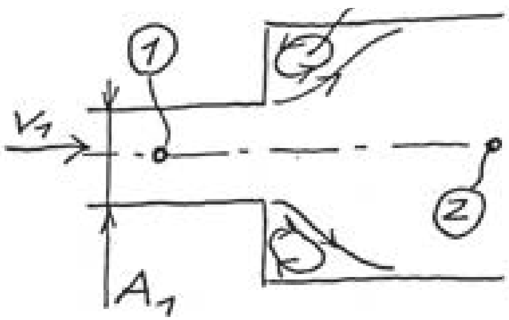


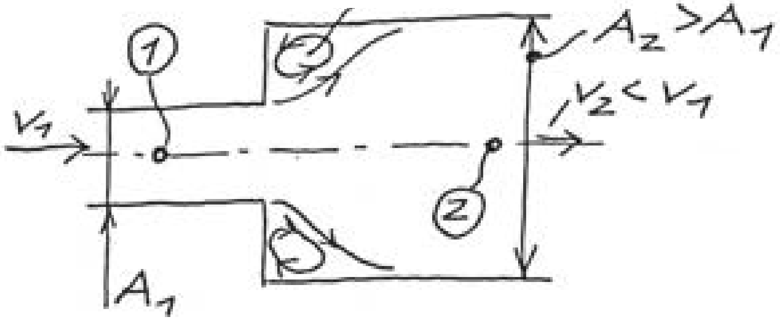
2.5.) Solution / description

- The more the n_{AR} → the more the deceleration ($v_2 < v_1$) → the more the potential for pressure recovery due to reducing the dynamic pressure: "ideal pressure rise": $\rho\left(\frac{v_1^2}{2} - \frac{v_2^2}{2}\right)$
- The less the n_{AR} → the less drastic change in flow path → reduced stagnation zone → moderation in $\Delta p'$ deteriorating the pressure recovery

? *Any ideas ?*

How to find an intermediate compromise in n_{AR} for maximum c_p?

$$\Delta p = \rho\left(\frac{v_1^2}{2} - \frac{v_2^2}{2}\right) - \Delta p' \quad (2.2b)$$



"Borda – Carnot equation"
"Sudden expansion of a pipe"
(Internet, e.g. Wikipedia)

$$\Delta p' = \frac{\rho}{2}(v_1 - v_2)^2 \qquad (2.5)$$

(2.2b)(2.5) →

$$\Delta p = \underbrace{\rho\left(\frac{v_1^2}{2} - \frac{v_2^2}{2}\right)}_{\text{ideal pressure rise}} - \underbrace{\frac{\rho}{2}(v_1 - v_2)^2}_{\Delta p'} =$$

$$= \frac{\rho}{2}\left\{v_1^2 - v_2^2 - (v_1 - v_2)^2\right\} =$$

$$= \frac{\rho}{2}\left\{(v_1 + v_2)(v_1 - v_2) - (v_1 - v_2)(v_1 - v_2)\right\} =$$

$$= \frac{\rho}{2}(v_1 - v_2)\left\{(v_1 + v_2) - (v_1 - v_2)\right\} =$$

$$= \frac{\rho}{2}(v_1 - v_2)\cdot 2 v_2 = \Delta p \qquad (2.6)$$

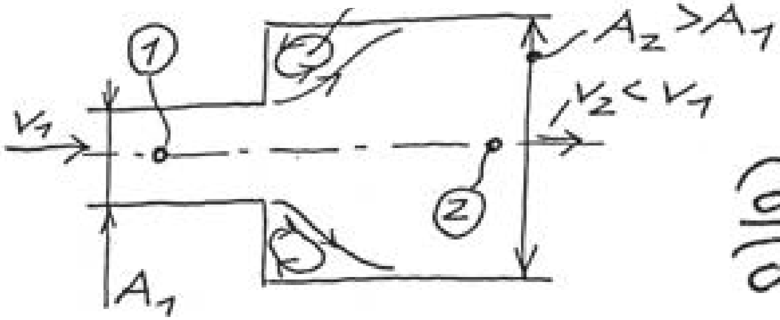
$$C_p = \frac{\Delta p}{\rho \frac{v_1^2}{2}} = \frac{\cancel{\frac{\rho}{2}}\cdot 2(v_1 - v_2)\cdot v_2}{\cancel{\frac{\rho}{2}}\, v_1^2} =$$

$$= 2\cdot\left[1 - \frac{v_2}{v_1}\right]\cdot\frac{v_2}{v_1} = C_p \qquad (2.7)$$

(2.7)(2.4a) →

$$C_p = 2\cdot\left[1 - \frac{1}{n_{AR}}\right]\cdot\frac{1}{n_{AR}} \qquad (2.8)$$

? ***How to find an n_{AR} to maximize C_p ?***



$$\frac{\partial c_p}{\partial n_{AR}} = 2\cdot\left\{\left[\frac{1}{n_{AR}^2}\cdot\frac{1}{n_{AR}}\right] + \left[1-\frac{1}{n_{AR}}\right]\cdot\frac{(-1)}{n_{AR}^2}\right\} =$$

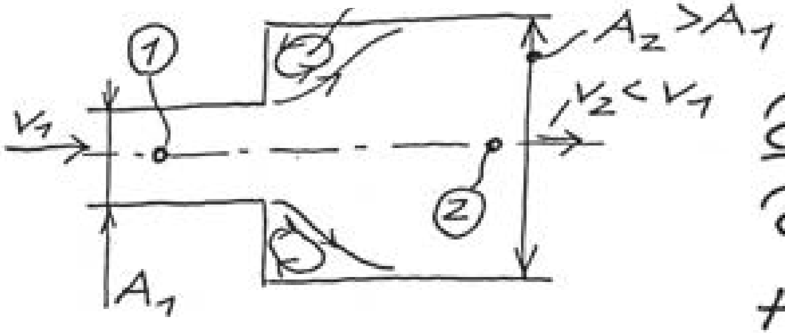
$$= 2\cdot\left\{\frac{1}{n_{AR}^3} - \frac{1}{n_{AR}^2} + \frac{1}{n_{AR}^3}\right\} =$$

$$= 2\cdot\left\{\frac{2}{n_{AR}^3} - \frac{1}{n_{AR}^2}\right\} = \frac{2}{n_{AR}^2}\left\{\frac{2}{n_{AR}} - 1\right\} \qquad (2.9)$$

$$\frac{\partial c_p}{\partial n_{AR}} := 0 \; ; \quad \frac{2}{n_{AR}} = 1$$

$$n_{AR} = 2 \qquad (2.10)$$

? ***What is missing ?***



$$\frac{\partial^2 c_p}{\partial n_{AR}^2} = + \frac{2\cdot(-2)}{n_{AR}^3}\left\{\frac{2}{n_{AR}} - 1\right\} + \frac{2}{n_{AR}^2}\left\{-\frac{2}{n_{AR}^2}\right\} \qquad (2.11a)$$

$@\, n_{AR} = 2:$

$$-\frac{4}{2^3}\underbrace{\left\{\frac{2}{2} - 1\right\}}_{\emptyset} + \frac{2}{2^2}\left\{-\frac{2}{2^2}\right\}$$

$$\left.\frac{\partial^2 c_p}{\partial n_{AR}^2}\right|_{@\, n_{AR}=2} < 0 \;\checkmark \qquad (2.11b)$$

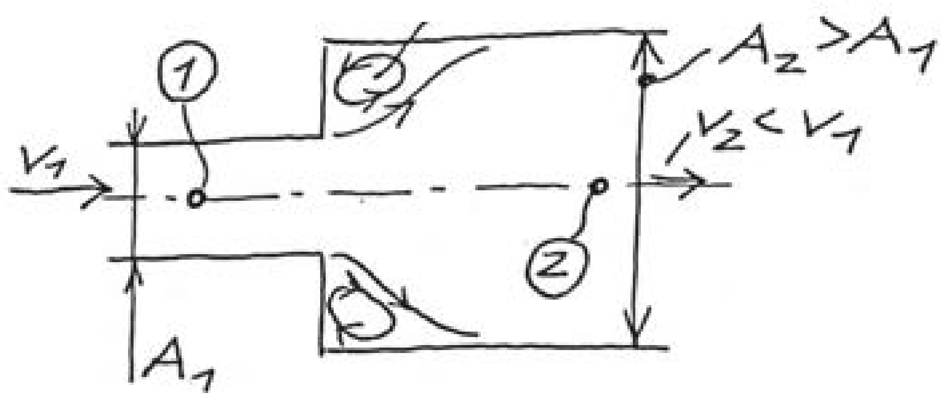
maximum!

? *Any issues with notation ∂ / ∂… ?*

n_{AR} **= 2 for** $C_{p\ max}$ **= 0,5**

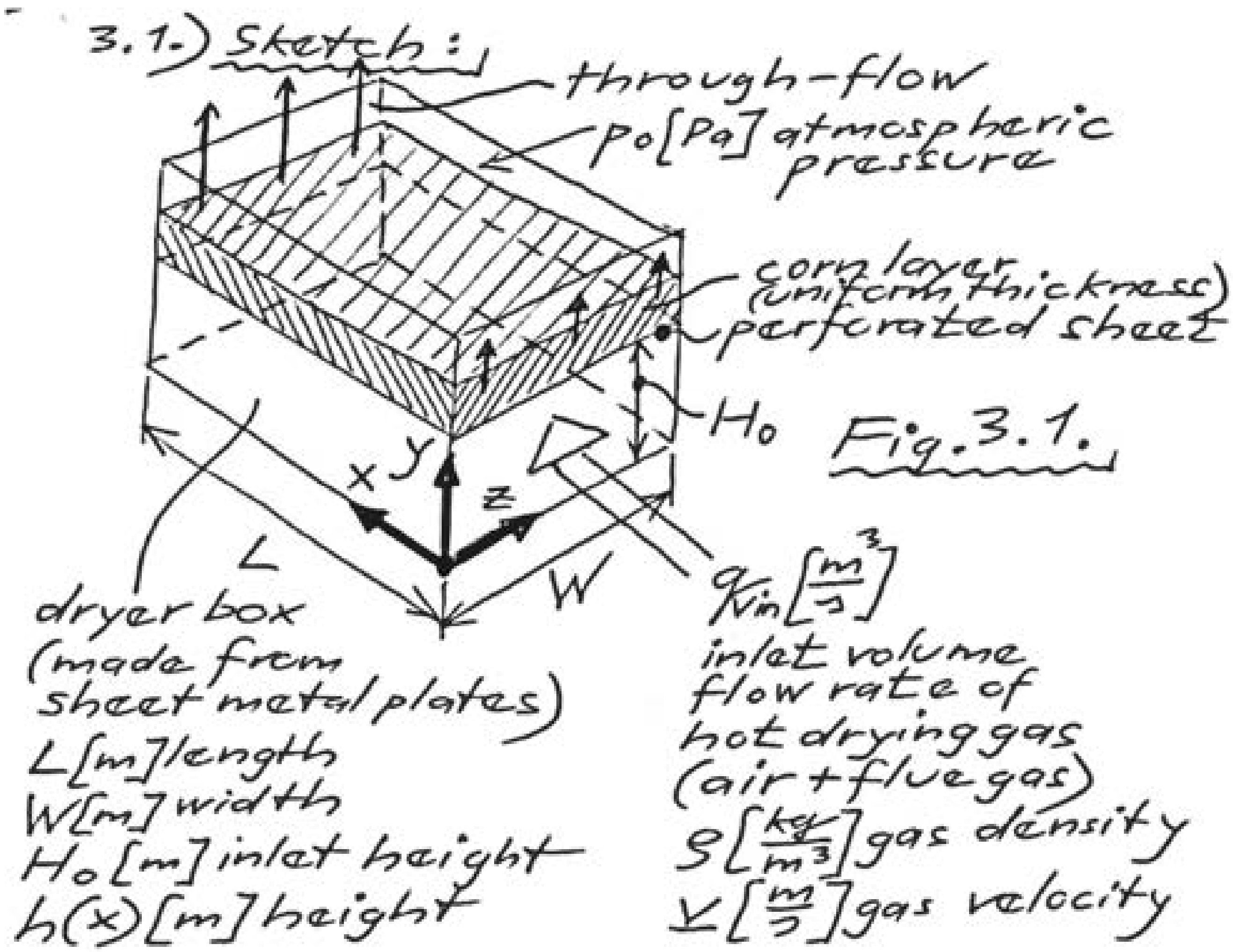
? *Any historical example for a similar practical analytical solution of such complexity ?*

III. CASE STUDY 2: CLEAN ROOM FOR MOBILE PHONE PRODUCTION



- $n_{AR} = A_2 / A_1 = 2$ for the maximum of specific pressure recovery, resulting in $C_p = 0,5$
- The half of inlet dynamic pressure (~ inlet kinetic energy) is converted to pressure rise
- Truly analytical solution, confirmed by experimental data (own and literature measurements)
- Sudden pipe expansions have been existing for centuries.
- Mathematical analysis has been existing for centuries.
- We have published this analytical solution first, in 2023!!! (*Lukács and Vad, 2023*)
- https://pp.bme.hu/me/article/view/22389/9863 – Publication of detailed interpretation is in progress
- The World is small.

Dr. János VAD, vad.janos@gpk.bme.hu



? *What machinery delivers the gas ?*

? *Combustion of what fuel, to obtain a „healthy" flue gas ?*

Dr. János VAD, vad.janos@gpk.bme.hu

If the shape of the drying box is a rectangular cuboid:

3.2. Formulation of problem:

$h(x) = \text{constant} \equiv H_0$ ↯ (3.1)

is BAD! non-uniform drying!

y-wise outflow through the corn layer:

- Near $x = 0$: "not enough": "underdrying"
- Near $x = L$: "too much": "overdrying" → corn burn-out

How to set the longitudinal (x-wise) distribution of $h(x)$ for constant (uniform) y-wise through-flow?

3.3. Assumptions:

i) $\rho = \text{constant}$ (incompressible) (3.2a)

ii) $v_z \equiv 0$ } 2D (planar) flow (3.2b)

iii) $\partial/\partial z \equiv 0$ } (3.2c)

iv) Wall flow friction neglected

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3.4.) Solution/description:

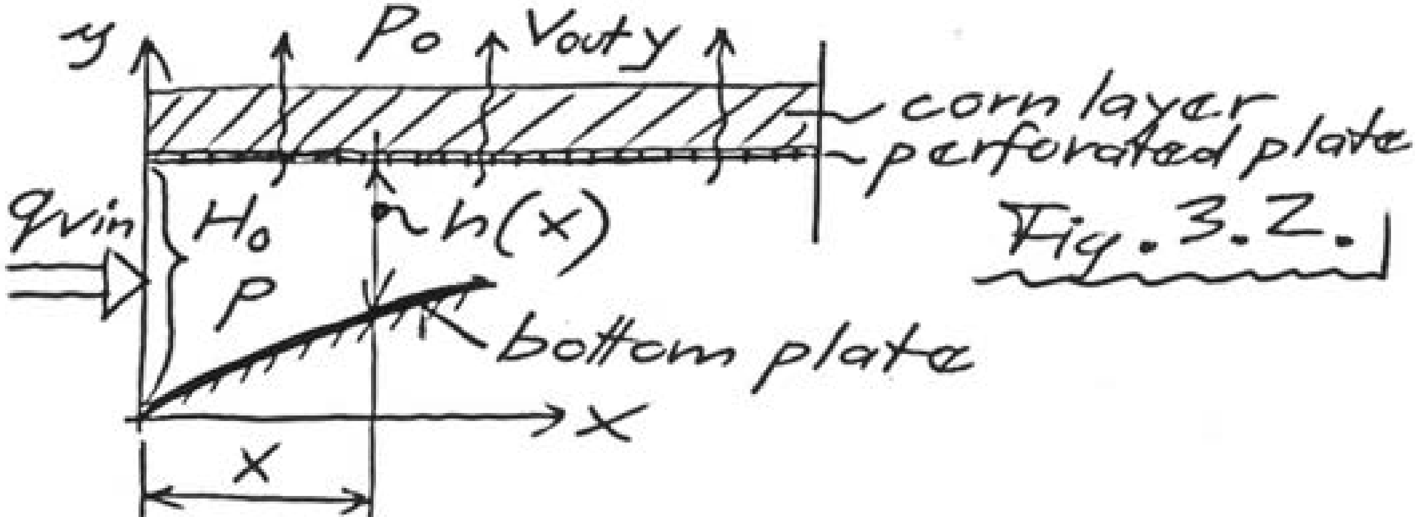

Fig. 3.2.

p [Pa]: internal pressure

$v_{out\,y}$ [m/s]: outflow velocity

$$p - p_0 \hat{=} \Delta p \text{ [Pa] overpressure} \quad (3.3)$$

Dr. János VAD, vad.janos@gpk.bme.hu

[Analogy with electricity:

Ohm's Law:

$$I = \frac{1}{R} \cdot \Delta U \qquad (3.4a)$$

current [C/s = A]

resistance

permeability

CONSTANT

voltage difference [V]

$$I \sim \Delta U \,. \qquad (3.4b)$$]

$$vouty \sim \Delta p \qquad (3.4c)$$

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Uniform through-flow:

$v_{out\,y} \equiv \text{constant} \rightarrow (3.5a)(1.7):$ (3.5a)

$$v_{out\,y} = \frac{q_{v\,in}}{A_{layer}} = \frac{q_{v\,in}}{L \cdot W} \qquad (3.5b)$$

$(3.4c) \rightarrow$

$\Delta p(x) \equiv \text{constant}$ (3.5c)

$p(x) \equiv \text{constant}$ (3.5d)

How to obtain? $h(x) = ?$ (3.5e)

$\rightarrow p(x) \equiv \text{constant}$: NO x-wise aerodynamic force is generated

? ***What principle (law) will be used ?***

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Newton II:

NO x-wise aerodynamic force:
NO x-wise change in fluid momentum should be generated!
"Conservation of momentum"

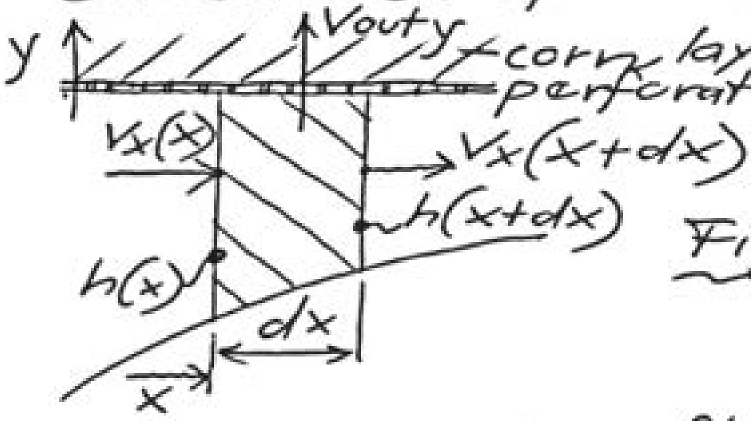

Fig. 3.3.

x-wise momentum flux (1.8):

$$\dot{I}_x = q_{mx} \cdot v_x = \underbrace{\{\varrho \cdot [\underbrace{h(x) \cdot W}_{A}] \cdot v_x\}}_{q_m(x)} \cdot v_x =$$

$$= \overbrace{\varrho \cdot W}^{const.} \cdot h(x) \cdot v_x^2(x) \qquad (3.6a)$$

$:= \text{constant!}$

? ***What additional principle (law) is ALWAYS available ? Conservation of…***

$$\frac{d\dot{I}_x}{dx} := 0 \quad (3.6a) \rightarrow \qquad (3.6b)$$

$$\frac{dh}{dx} \cdot v_x^2 + h \cdot 2v_x \cdot \frac{dv_x}{dx} = 0 \quad (3.6c)$$

Additional equation – ALWAYS!
Conservation of fluid mass:
x-wise volume flow rate:

$$q_{vx} = v_x \cdot \underbrace{h \cdot W}_{A} \quad (3.7a)$$

$$-dq_{vx} = v_{out\,y} \cdot \underbrace{dx \cdot W}_{dA_{layer}} \quad (3.7b)$$

$$-\frac{dq_{vx}}{dx} = v_{out\,y} \cdot W \quad (3.7c)$$

$(3.7a)(3.7c) \rightarrow$

$$-\cancel{W}\left(\frac{dv_x}{dx}h + v_x\frac{dh}{dx}\right) = v_{out\,y} \cdot \cancel{W} \quad (3.7d)$$

Dr. János VAD, vad.janos@gpk.bme.hu

(3.6c) →

$$\frac{dv_x}{dx} = -\frac{dh}{dx}\frac{v_x^2}{h \cdot 2v_x} = -\frac{1}{2h}\frac{dh}{dx}v_x$$

(3.7e)

(3.7d)(3.7e) →

$$-\left[-\frac{1}{2h}\frac{dh}{dx}v_x h + v_x\frac{dh}{dx}\right] = v_{out\,y} =$$

$$= -\frac{1}{2}\frac{dh}{dx}v_x = v_{out\,y} \qquad (3.8)$$

But

$$q_{vx}(x) = v_x(x) \cdot \underbrace{h(x) \cdot W}_{A} = q_{v\,in} - v_{out\,y} \cdot x \cdot W$$

(3.9a) (3.9a)

(3.5b) →

$$v_x \cdot h \cdot W = q_{v\,in} - \underbrace{\frac{q_{v\,in}}{L \cdot W}}_{v_{out\,y}} \cdot x \cdot W = q_{v\,in}\left[1 - \frac{x}{L}\right]$$

(3.9b)

$$v_x = \frac{q_{v\,in}}{h \cdot W}\left[1 - \frac{x}{L}\right] \qquad (3.9c)$$

Dr. János VAD, vad.janos@gpk.bme.hu

(3.8)(3.9c)(3.5b) $\rightarrow$

$$-\frac{1}{2}\frac{dh}{dx}\frac{\cancel{q_{v\,in}}}{h\cdot W}\left[1-\frac{x}{L}\right]=\frac{\cancel{q_{v\,in}}}{L\cdot W} \quad (3.10a)$$

Solution: independent from $q_{v\,in}$!

ROBUST!

$$-\frac{1}{2}\frac{1}{h}\frac{dh}{dx}\left[1-\frac{x}{L}\right]=\frac{1}{L} \quad (3.10b)$$

$$X \hat{=} \frac{x}{L} \quad (3.11a)$$

$$H \hat{=} \frac{h}{H_0} \quad (3.11b)$$

(3.10b)(3.11a)(3.11b) $\rightarrow$

$$-\frac{1}{2}\frac{1}{H}\frac{dH}{dX}\left[1-X\right]=1 \quad (3.12a)$$

$$-\frac{1}{2}\frac{dH}{H} = \frac{d\overline{X}}{1-\overline{X}} \qquad (3.12b)$$

$$\int_1^H \frac{dH}{H} = -2\int_0^{\overline{X}} \frac{d\overline{X}}{1-\overline{X}} \qquad (3.12c)$$

$$\ln H = 2\ln(1-\overline{X}) \qquad (3.12d)$$

$/ \exp$

$$\underline{H = (1-\overline{X})^2} \qquad (3.13)$$

Dr. János VAD, vad.janos@gpk.bme.hu

Tangents :

$$\frac{dH}{d\bar{X}} = 2(1-\bar{X})(-1) \qquad (3.14a)$$

$$@\ \bar{X}=0 : \frac{dH}{d\bar{X}} = -2 \qquad (3.14b)$$

$$@\ \bar{X}=1 : \frac{dH}{d\bar{X}} = 0 \qquad (3.14c)$$

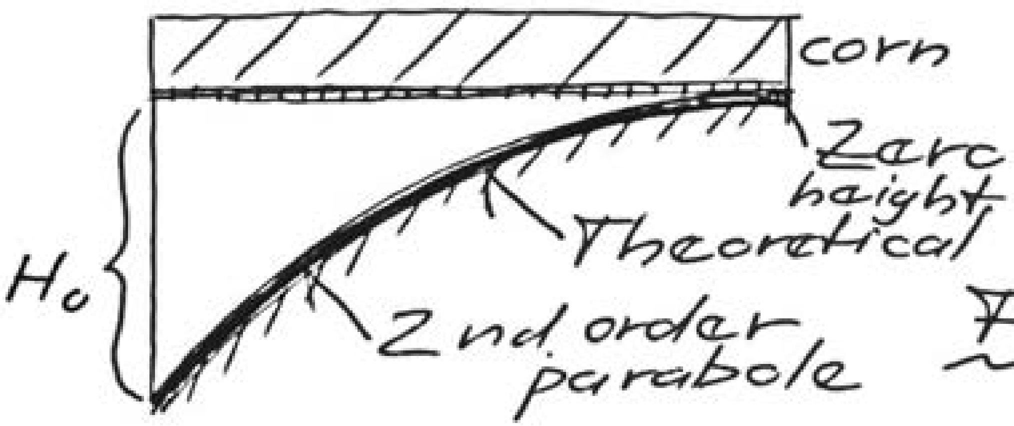

Fig. 3.4.

?

What approximation would you recommend for easy and cost-effective manufacturing ?

3.5.)

Practical approximation:

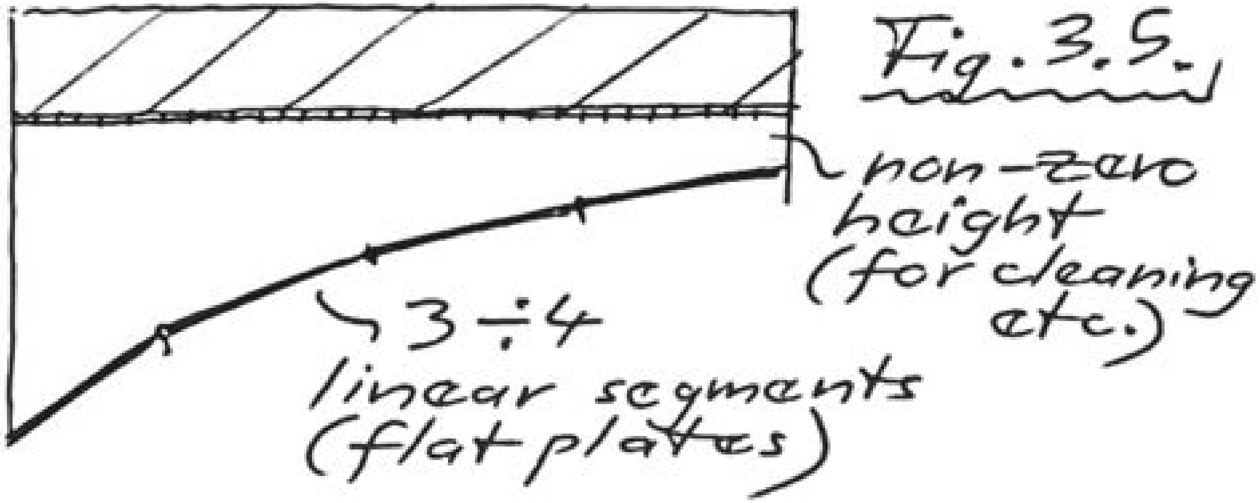

Fig. 3.5.

Dr. János VAD, vad.janos@gpk.bme.hu

V. SUMMARY: KEEP IT SIMPLE → AND MOVE FORWARD UPON DEMAND

Case study 1: Watermill in the Danube; turbine principle – WATER

- Complex 3D flow (← CFD…) in the vicinity of the rotor blades ← simplifications
- Order-of-magnitude estimations; confirmation / rebuttal of the feasibility of operational / design concepts
- **Mathematics tools: basic operations (multiplication, division, addition, extraction) → mental arithmetic; extreme analysis by means of derivation**

Case study 2: Clean room technology for mobile phone production – AIR

- Complex 3D flow in the stagnant zone (separation bubble) of the expansion ← simplifications
- Purposeful preliminary design of a parameter: area ratio for maximum pressure recovery
- **Mathematics tools: basic operations; extreme analysis by means of derivation**

Case study 3: Corn drying technology – HOT FLUE GAS + AIR MIXTURE

- Complex 3D flow in the vicinity of the corn gravels (within the layer) ← simplifications
- Purposeful preliminary design of a distribution: $h(x)$ for uniform through-flow
- **Mathematics tools: basic operations; derivation; ordinary differential equation; integration**

Dr. János VAD, vad.janos@gpk.bme.hu

YOUR QUESTIONS ARE WELCOME

Simple maths for complex flows

THANK YOU FOR YOUR ATTENTION AND CONTRIBUTION!
Keep in touch: vad.janos@gpk.bme.hu