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http://www.math.bme.hu/tamop422
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ABSTRACTS OF TALKS:

Jean-Paul André (BME Graduate School of Mathematics and Computer Science):

Modeling stone erosion.

Erosion of stone is a problem of surface evolution resulting in a class of geometric PDEs. The speed of erosion is the main parameter of the process and usually contains three terms: a constant term, a term proportional to the main curvature and a term proportional to the Gaussian curvature. Numerically, the level set method has turned out to be the most suited for this problem because of its capability to handle geometrical changes. Although, the expression of the speed suggests a separated study of its terms that could be achieved with the so called operator splitting techniques. Those accurate simulations would allow us to study the limit shapes of the volume before disappearing and experiment new speeds of erosion. This presentation proposes an analysis of the solutions of the PDE under some simplifying conditions. Matlab simulations will be also presented.

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Balázs Bárány (BME Graduate School of Mathematics and Computer Science):

Slicing the Sierpinski gasket.

(Joint work with Károly Simon and Andrew Ferguson)

We investigate the dimension of intersections of the Sierpinski gasket with lines. First, we prove a dimension conservation principle, which connects the local dimension of the projected natural measure and the box dimension of the slices. We show that the slices with rational slope are exceptional in Marstrand's theorem. Precisely, the dimension of a Lebesgue-typical slice with rational slope is strictly less then the dimension of the gasket minus one and respectively to the natural self-similar measure, the dimension of a typical slice with rational slope is strictly greater then the dimension of the gasket minus one. On the other hand, we provide a multifractal analysis for the set of points in the projection for which the associated slice has a prescribed dimension.

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Zsuzsanna Barta (BME Graduate School of Mathematics and Computer Science):

The locomotive assignment problem.

There are many optimization problems in connection with railway operation and they are sufficiently complex and almost exclusively large tasks. The larger railway companies nowadays employ matematical researchers so as to be cost-effective.

In the presentation we discuss a particular task of the railway optimization problem, the locomotive assignment problem. The problem is the following: for a fix period (for example a day or a week) a region's timetable is known and the goal is to assign minimum number of locomotives to the trains. Solving this particular problem can take up to several days for the transportation engineers. We study such matematical models which can describe other needs of the railway companies (e. g. various types of locomotives, repeatability conditions).

The topic is very timely because we work together with the MÁV-Trakció company to create the matematical background of a computer-based decision-support system in order to help the dispatcher's work. The company give engine-drivers and/or locomotives to the incoming requests for freight trains. In this project the most of our existing results we can use, but the models have to take notice of some other conditions. There is a need such matematical models which can be solved real-time. There are a lot of requests and the company doesn't have enough locomotives, so there is a need for light travel locomotives and we have to deal with the services of the locomotives. The procurer companies often decline their request and sometimes they ask resources (engine-driver and/or locomotive) for several different time for the same freight train, because the cost of renouncement is very low. Further problem is the large delays of the freight trains. It is true especially for international freight trains. We have to deal with this information too which is an ongoing work.

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Lajos Boróczki (BME Graduate School of Mathematics and Computer Science):

Face and edge transitive tilings in three dimensional non-Euclidean spaces.

Any tiling, generated by a crystallographic group with compact fundamental domain, can be represented by a diagram and a matrix valued function, based on their barycentric subdivision and the adjacency relations between the orbits and the particular simplices. The representation is called the D-symbol of a tiling in honour of Delone, Delaney and Dress. The representation is easily adaptable to computer programs.

Both face and edge transitive tilings are special tilings, which have only 1 face or edge orbit respectively in the crystallographic group. (Our fundamental domain doesn't have to be the smallest possible, it can have inner symmetries.) It's easy to prove that there are finitely many possible diagrams in these cases, so enumerating all of them in every possible geometry seems possible.

The problem in the non-Euclidean planes (E. Molnár with Z. Lučić and M. Stojanović (1994)) and in the space E^3 (E. Molnár with A.W.M. Dress and D.H. Huson (1993)) is already solved.

We can enumerate tilings with at most 18 simplex-orbits (out of 24). Three dimensional tilings induce some two dimensional tilings around the vertices of the simplices and between any two partition of the simplex-vertices. These can be examined by a simple formula, which gives us finitely many possible matrix valued functions for every diagram to investigate.

Based on the Thurston-theorem there are 8 possible geometries. There exists at least 4 proof of the theorem but none of them is constructive. Based on our method it would be easier to find a tiling which does not fit in any of the 8 geometries; but inspecting the tilings we can possibly move forward to a constructive proof of the theorem.

But there are some more problems to solve first: we have to find the possible splittings in a D-symbol (which corresponds to tori in Thurston's notation) and we have to be able to tell the signature of the projective space of the tiling of a primitive D-symbol.

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Rudolf Csikja (BME Graduate School of Mathematics and Computer Science):

Dynamical systems with hysteresis.

We consider a planar piecewise affine ODE with two parameters. A full bifurcation analysis of the system is given by investigating the corresponding two-valued Poincaré-map. As a generalization, we investigate one-dimensional piecewise linear interval maps with hysteresis. We present a construction of invariant density functions for the hysteretic version of two classical maps; the beta map and the tent map.

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Balázs Csizmadia (BME Graduate School of Mathematics and Computer Science):

Almost shortest paths in $O(n^{\frac{3}{2}}m^{\frac{1}{2}})$ time.

In this talk, I present an algorithm to compute all pairs almost shortest paths in an unweigted, undirected graph with additive error of at most 2. The algorithm runs in $O(n^{\frac{3}{2}}m^{\frac{1}{2}})$ time. In the talk I show the steps of the algorithm, describe and how it works. There is a similar algorithm, that runs in $O(n^{\frac{7}{3}})$ time. These algorithms are improvements on existing algorimthms by a polylogarithmic factor.

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Julianna Göbölös-Szabó (BME Graduate School of Mathematics and Computer Science):

Link Prediction for Multilingual Wikipedia.

(Joint work with Natalia Prytkova, Marc Spaniol and Gerhard Weikum)

Knowledge bases like Wikipedia are edited in many different languages by independent communities. Editors aspire to make the versions of different languages mutually consistent, by using interwiki links between identical entities. However, the current status is far from achieving this goal: the sizes of different versions are divergent, and the link structures drastically differ across culturally specific editions.

Our tool, LAIKA, helps Wikipedia editors to add new links or content to one regional version of Wikipedia using other versions in different languages, and trying to identify meaningful mappings between their nodes. In this work, we focus on cleansing and unifying the category hierarchy of the different languages by finding interwiki links between categories and recommending new categories for articles. In addition, we consider also finding related entities and external references, and comparing them across language editions.

The main issues we faced during link prediction for cross-lingual Wikipedia are the selection of potential link neighbors and ranking them by similarity using random-walk techniques and combinatiorical approaches. LAIKA is an interactive online tool. Thus, we need fast algorithms for both problems that are applicable to graphs with millions of nodes and tens of millions of edges.

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Illés Horváth (BME Graduate School of Mathematics and Computer Science):

Newer versions of sector conditions.

(Joint work with Bálint Tóth and Bálint Vető)

The theory of central limit theorems for additive functionals of Markov processes via martingale approximation was initiated in 1986 by Kipnis and Varadhan. Since then, the theory has been widely extended by Varadhan and others; so-called sector conditions were introduced in order to provide sufficient conditions that can be checked directly in particular applications. Applications include different types of random walks in random environments and random walks with long memory, e.g. the tagged particle in simple exclusion, persistent random walk in random environment, the myopic self-avoiding walk or the self-repellent Brownian polymer.
We provide a newer version of the graded sector condition; apart from the conditions being less restrictive than in previous formulations, the proof is also less technical.

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Tamás Kiss (BME Graduate School of Mathematics and Computer Science):

SVD factorization methods for computing missing matrix entries

Matrix factorization technics is a key part in predicting missing values of large rating matrices. These technics are used in search engine advertising, collaborative filtering and recommender systems, therefore are very important for commerce. In practical problems the training matrices are large so standard factorization methods like the Lanczos algorithm cannot be applied. Regularized singular value decomposition was introduced in the Netflix prize competition which computes latent vectors for users and items providing estimation for missing values namely the scalar product of the user and item feature vectors. Computing feature vectors is done by gradient method. Another key part of the previous model is the regularization technic which helps avoiding overfitting. The work shows how these methods work on the kddcup2012 task 2 which is a well-known data mining competition. In the previous task additional features like hierarchical information are given so the above mentioned model should be combined with other technics.

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Tamás Kói (BME Graduate School of Mathematics and Computer Science):

Capacity Regions of Partly Asynchronous Multiple Access Channels.

(Joint work with Lóránt Farkas)

Multiple access channels (MACs) describe the situation when many senders send messages to one receiver simultaneously. MACs are most frequently studied under the assumption that the senders cannot communicate with each other but are able to maintain frame synchronism. An asynchronous MAC (AMAC) arises when this assumption fails, causing unknown delays between the starting times of the codewords of the different senders. Here a single letter characterization is given for the capacity region of discrete memoryless partly asynchronous multiple access channels (PAMACs). These are AMACs with the senders divided into groups, the senders belonging to the same group are synchronized but the groups are not synchronized with each other.

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Júlia Komjáthy (BME Graduate School of Mathematics and Computer Science):

First passage percolation on inhomogeneous random graphs.

We investigate first passage percolation (FPP) on inhomogeneous random graphs (IHRG). The random graph model $G(n, \kappa)$ we first study is the finite type case of the general model introduced by Bollobas, Janson and Riordan where edges between vertices are drawn independently with probabilities depending on the type of the vertices. Additionally, each existing edge is given an independent exponential edge weight with parameter 1. Afterwards the general type spaces are also considered in the case where each type has the same average number of neighbors.
We generalize results of Bhamidi, van der Hofstad and Hooghimstra, where FPP is explored on the Erdos-Renyi random graphs, a special case where all the vertices are of the same type. We find analogous results for the minimal weight of the path between uniformly chosen vertices in the giant component and for the hopcount, i.e. the number of edges on this minimal weight path.

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Gábor Lovics (BME Graduate School of Mathematics and Computer Science):

A method to approximate the Pareto-optimal set of the Markowitz-model.

(Joint work with Tibor Illés)

In economics and financial literature the measure of risk was always a very interesting topic, and nowadays it gets a lot of attention. One of the first idea to take into consideration the risk in financial activities, came from Harry Markowitz who developed his famous model where the investors make portfolios from different securities, and try to maximize their profit and minimize their risk at the same time. In this model the profit was linear and the risk was defined as the variance. From mathematical programming point of view the Markowitz-model can be formulated as linearly constrained optimization problem with two objective (linear profit and quadratic) functions. In this type of problem we need to find solutions, where one of the objectives can not be improved without worsen the other. These solutions are called Pareto-optimal solutions.

The original idea was to solve the Model by summing the two objective functions, with given weights, but with this method only one Pareto-optimal solution can be found and we lost a lot of important information. For approximation of the Pareto-optimal set, we use feasible joint decrease directions, and generalize the subdivision algorithm developed by Olives Schütze et al..

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Péter Móra (BME Graduate School of Mathematics and Computer Science):

Multifractral analysis of the TCP protocol.

(Joint work with Júlia Komjáthy, Károly Simon and Sándor Molnár)

The TCP (Transmission Control Protocol) is a network protocol, which establishes reliable connection between two nodes. Since 2006 the CUBIC implementation is used by default in the linux kernel, which uses the following rate to send:

$$C\left(t-\sqrt[3]{\frac{w\beta}{C}}\right)^3+w$$

where $C>0$, $0<\beta<1$ are parameters, w is the sending rate just before the last loss occurred, and t is the time left since the last error. We consider the multifractal analysis of this flow control mechanism.

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Péter Nándori (BME Graduate School of Mathematics and Computer Science):

Lorentz process with shrinking holes in a wall.

(Joint work with Domokos Szász)

We acertain the diffusively scaled limit of a periodic Lorentz process in a strip with an almost reflecting wall at the origin. Here, almost reflecting means that the wall contains a small hole vanishing in time. The limiting process is a quasi-reflected Brownian motion, which is Markovian but not strong Markovian. Local time results for the periodic Lorentz process, having independent interest, are also found and used.

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András Némedy Varga (BME Graduate School of Mathematics and Computer Science):

Investigating statistical properties of higher dimensional hyperbolic systems by coupling techniques.

In order to handle small perturbations in planar dispersing billiards Chernov and Dolgopyat developed a technique to investigate statistical properties of two dimensional hyperbolic dynamical systems that is based on the coupling of standard pairs. These pairs are unstable curves, with sufficiently regular measures on them. When compared with other successful methods – in particular Young's tower construction, which was the first technique that could be used to prove exponential decay of correlations for a class of multidimensional dispersing billiards – the method does not use symbolic coding, hence it is more geometric and more suitable for handling perturbative aspects. The standard pair technique has turned out to be very powerful, however, as of yet, it is restricted to two-dimensional systems. In this talk I would like to report on my work in progress concerning the extension of the standard pair technique to higher dimensions, explaining the difficulties arising from the high dimensional setup. These include in particular proving the regularity of the high dimensional holonomy map, extensions of certain dynamically Hölder continuous functions from Cantor-sets to hypersurfaces keeping their regularity properties, and the non-trivial construction of the coupling time function.

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Eszter Rozgonyi (BME Graduate School of Mathematics and Computer Science):

Additive representation functions.

(Joint work with Sándor Z Kiss and Csaba Sándor)

Let $\mathbb{N}$ be the set of nonnegative integers. For a given set $\mathcal{A} \subset \mathbb{N}$ the representation functions $R^{(1)}_{h,\mathcal{A}}(n)$, $R^{(2)}_{h,\mathcal{A}}(n)$ and $R^{(3)}_{h,\mathcal{A}}(n)$ are defined as the number of solutions of the equation $a_{i_1} + \cdots + a_{i_h} = n$, $a_{i_1}, \dots, a_{i_h} \in \mathcal{A}$ without any condition, with condition $a_{i_1} < \cdots < a_{i_h}$ and $a_{i_1} \leq \cdots \leq a_{i_h}$, respectively.
In 1978, Nathanson proved that if $\mathcal{A}$ and $\mathcal{B}$ are distinct nonempty sets of integers such that $R^{(1)}_{2,\mathcal{A}}(n) = R^{(1)}_{2,\mathcal{B}}(n)$ for all sufficiently large n then $\mathcal{A} = F_{\mathcal{A}} \cup S$ and $\mathcal{B} = F_{\mathcal{B}} \cup S$, where $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S have some nice properties. We extend this result to the $R^{(1)}_{3}(n)$ representation function. In this case the construction of the sets $F_{\mathcal{A}}, F_{\mathcal{B}}$ and S is a bit difficult but still nice.
In 2011, Yang posed the following problem in his article: if $p \geq 3$ is a prime and $\mathcal{A}$ is a set of nonnegative integers, then does there exist a set of nonnegative integers $\mathcal{B}$ with $\mathcal{A} \neq \mathcal{B}$ such that $R^{(1)}_{p,\mathcal{A}}(n) = R^{(1)}_{p,\mathcal{B}}(n)$ for all sufficiently large n? We solved this problem in a generalized way. We proved that for arbitrary $h \geq 2$, $h \in \mathbb{Z}$ there exist $\mathcal{A}, \mathcal{B} \subset \mathbb{N}$, $\mathcal{A} \neq \mathcal{B}$, such that $R^{(i)}_{h,\mathcal{A}}(n) = R^{(i)}_{h,\mathcal{B}}(n)$ for all sufficiently large n and for $i = 1, 2, 3$.
In the proofs we use generating functions and some other theorems from algebraic and analytic number theory.

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László Ruppert (BME Graduate School of Mathematics and Computer Science):

Efficient state estimation for quantum systems.

We examined different state estimation scenarios and gave the best estimation schemes. We considered the case of multiple von Neumann measurements as well as that of a single POVM measurement. We analyzed the problem of partial a priori information for qubits and multi-level systems as well. As a special case, we also considered the setup when no information is known beforehand. We introduced a new generalization of SIC-POVMs and examined its properties both analytically and numerically.

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Péter Szabó (BME Graduate School of Mathematics and Computer Science):

Bounds on the Number of Edges in Hypertrees.

Let $\mathcal{H}$ be a k-uniform hypergraph. A chain in $\mathcal{H}$ is a sequence of its vertices such that every k consecutive vertices form an edge. In 1999 Gyula Y. Katona and Hal Kierstead suggested to use chains in hypergraphs as the generalisation of paths. Although a number of results have been published on Hamilton-chains in recent years, the generalisation of trees with chains has still remained an open area.

We generalise the concept of trees for uniform hypergraphs. We say that a k-uniform hypergraph $\mathcal{F}$ is a hypertree if every two vertices of $\mathcal{F}$ are connected with a chain, and an appropriate kind of cycle-free property holds. An edge-minimal hypertree is a hypertree whose edge set is minimal with respect to inclusion.

After considering these definitions, we show that a k-uniform hypertree on n vertices has at least $n-(k-1)$ edges up to a finite number of exceptions, and it has at most $\binom{n}{k-1}$ edges. The latter bound is asymptotically sharp in the case of $k=3$. We give an upper bound on the edge-number of an edge-minimal hypertree and conjecture that $\frac{1}{k-1}\binom{n}{2}$ is an upper bound. We give a construction to show that if the conjecture is true then the inequality is sharp in asymptotic sense.

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András Szántó (BME Graduate School of Mathematics and Computer Science):

Complementarity in Matrix Algebras.

A concept in quantum information theory has led to the investigation of the subject. We consider two types of subalgebras of the complex matrix algebra $M_n \otimes M_n$: maximal abelian subalgebras (corresponding to measurements) and subalgebras isomorphic to M_n (corresponding to subsystems). Complementarity is a generalization of mutually unbiasedness as orthogonality of the traceless subspaces of the subalgebras. Among other results, we provide all possible direct-sum decompositions of the full matrix-algebra to pairwise complementary subalgebras in the $n=2$ case. The related idea of conditional POVMs are studied as well.

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Ágnes Tóth (BME Graduate School of Mathematics and Computer Science):

The ultimate categorical independence ratio.

The *independence ratio* of a graph G is defined as $i(G) = \frac{\alpha(G)}{|V(G)|}$, that is, as the ratio of the independence number and the number of vertices. For two graphs G and H, their *categorical product* (also called as direct product) $G \times H$ is defined on the vertex set $V(G \times H) = V(G) \times V(H)$ with edge set $E(G \times H) = \{\{(x_1, y_1), (x_2, y_2)\} : \{x_1, x_2\} \in E(G)$ and $\{y_1, y_2\} \in E(H)\}$. The kth categorical power $G^{\times k}$ is the k-fold categorical product of G. In 1996 Brown, Nowakowski and Rall defined the *ultimate categorical independence ratio* of a graph G as

$$A(G) = \lim_{k \to \infty} i(G^{\times k}).$$

They proved that for any independent set U of G the inequality $A(G) \geq \frac{|U|}{|U|+|N_G(U)|}$ holds, where $N_G(U)$ denotes the neighborhood of U in G. Furthermore, they showed that $A(G) > \frac{1}{2}$ implies $A(G) = 1$.

Let $a(G) = \max\{\frac{|U|}{|U|+|N_G(U)|} : U$ is an independent set of $G\}$. Alon and Lubetzy proposed the question whether $A(G) = a(G)$ if $a(G) \leq \frac{1}{2}$, and $A(G) = 1$ otherwise.

Recently, I managed to answer this question affirmatively. In the talk I am going to show the main ideas of the proof. I will also discuss some other open problems related to the ultimate categorical independence ratio which are immediately settled by this result. For instance, we obtain a proof for the conjecture of Brown, Nowakowski and Rall, stating that $A(G + H) = \max\{A(G), A(H)\}$, where $G + H$ denotes the disjoint union of the graphs G and H. In the proof I exploited an idea of Zhu that he used on the way when proving the fractional version of Hedetniemi's conjecture in 2011.

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Ambrus Zsbán (BME Graduate School of Mathematics and Computer Science):

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