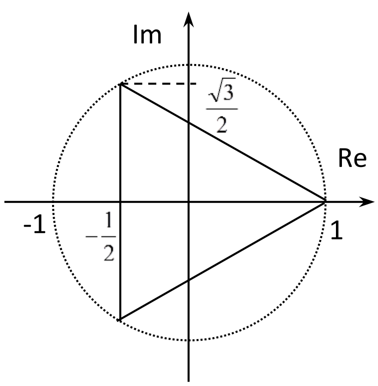
$\sqrt[3]{1} = ?$

$\sqrt[3]{1} = \sqrt[3]{1(\cos 0 + i\sin 0)} = \sqrt[3]{1}\left(\cos\frac{0+2\pi\cdot k}{3} + i\sin\frac{0+2\pi\cdot k}{3}\right)$, ahol k=0,1,2

$$k = 0 \;\Rightarrow z_1 = 1\left(\cos\frac{0+2\pi\cdot 0}{3} + i\sin\frac{0+2\pi\cdot 0}{3}\right) = 1(\cos 0 + i\sin 0) = 1(1 + i\cdot 0) = 1$$

$$k = 1 \;\Rightarrow z_2 = 1\left(\cos\frac{0+2\pi\cdot 1}{3} + i\sin\frac{0+2\pi\cdot 1}{3}\right) = 1\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$$

$$k = 2 \;\Rightarrow z_3 = 1\left(\cos\frac{0+2\pi\cdot 2}{3} + i\sin\frac{0+2\pi\cdot 2}{3}\right) = 1\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right) = -\frac{1}{2} - i\frac{\sqrt{3}}{2}$$



$$\sqrt[4]{-4} = \sqrt[4]{4(\cos\pi + j\sin\pi)} = \sqrt[4]{4}\left(\cos\frac{\pi+2k\pi}{4} + j\sin\frac{\pi+2k\pi}{4}\right)$$

$$k = 0 \quad\Rightarrow\quad z_1 = \sqrt{2}\left(\cos\frac{\pi}{4} + j\sin\frac{\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = 1 + j$$

$$k = 1 \quad\Rightarrow\quad z_2 = \sqrt{2}\left(\cos\frac{3\pi}{4} + j\sin\frac{3\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right) = -1 + j$$

$$k = 2 \quad\Rightarrow\quad z_3 = \sqrt{2}\left(\cos\frac{5\pi}{4} + j\sin\frac{5\pi}{4}\right) = \sqrt{2}\left(-\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = -1 - j$$

$$k = 3 \quad\Rightarrow\quad z_4 = \sqrt{2}\left(\cos\frac{7\pi}{4} + j\sin\frac{7\pi}{4}\right) = \sqrt{2}\left(\frac{\sqrt{2}}{2} - j\frac{\sqrt{2}}{2}\right) = 1 - j$$

$$\sqrt[3]{-2+2j}=\sqrt[3]{\sqrt{8}\left(\cos\frac{3\pi}{4}+j\sin\frac{3\pi}{4}\right)}=\sqrt{\sqrt[3]{8}}\left(\cos\frac{\frac{3\pi}{4}+2k\pi}{3}+j\sin\frac{\frac{3\pi}{4}+2k\pi}{3}\right)=$$

$$=\sqrt{2}\left(\cos\left(\frac{\pi}{4}+\frac{2k\pi}{3}\right)+j\sin\left(\frac{\pi}{4}+\frac{2k\pi}{3}\right)\right)$$

$$k=0 \quad\Rightarrow\quad z_1=\sqrt{2}\left(\cos\frac{\pi}{4}+j\sin\frac{\pi}{4}\right)=\sqrt{2}\left(\frac{\sqrt{2}}{2}+j\frac{\sqrt{2}}{2}\right)=1+j$$

$$k=1 \quad\Rightarrow\quad z_2=\sqrt{2}\left(\cos\frac{11\pi}{12}+j\sin\frac{11\pi}{12}\right)=\sqrt{2}\left(-\cos\frac{\pi}{12}+j\sin\frac{\pi}{12}\right)=-\frac{\sqrt{3}+1}{2}+j\frac{\sqrt{3}-1}{2}$$

$$k=2 \quad\Rightarrow\quad z_3=\sqrt{2}\left(\cos\frac{19\pi}{12}+j\sin\frac{19\pi}{12}\right)=\sqrt{2}\left(\sin\frac{\pi}{12}-j\cos\frac{\pi}{12}\right)=\frac{\sqrt{3}-1}{2}-j\frac{\sqrt{3}+1}{2}$$

Az eredmények meghatározásához a következőket vegyük figyelembe:

$$\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\sin\frac{\pi}{4}\cos\frac{\pi}{6}-\cos\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}-\frac{\sqrt{2}}{2}\frac{1}{2}=\frac{\sqrt{2}}{4}\left(\sqrt{3}-1\right)$$

$$\cos\frac{\pi}{12}=\cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=\cos\frac{\pi}{4}\cos\frac{\pi}{6}+\sin\frac{\pi}{4}\sin\frac{\pi}{6}=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\frac{1}{2}=\frac{\sqrt{2}}{4}\left(\sqrt{3}+1\right)$$