# Analízis 1, 14. hét

## Valós integrálok kiszámítása

I[A] . Igazoljuk az alábbi egyenlőségeket.

1. $\displaystyle\int_0^\infty \frac{x^2}{(x^2+1)(x^2+4)}\,\mathrm{d}\,x = \frac{\pi}{6}$
2. $\displaystyle\int_0^\infty \frac{1}{(x^2+2x+2)(x^2-2x+2)}\,\mathrm{d}\,x = \frac{\pi}{8}$
3. $\displaystyle\int_0^\infty \frac{1}{x^4+1}\,\mathrm{d}\,x = \frac{\pi}{2\sqrt{2}}$
4. $\displaystyle\int_0^\infty \frac{1}{(x^2+1)^2}\,\mathrm{d}\,x = \frac{\pi}{4}$
5. $\displaystyle\int_0^\infty \frac{1}{(x^2+4)^3}\,\mathrm{d}\,x = \frac{3\pi}{512}$
6. $\displaystyle\int_0^\infty \frac{x^2}{(x^2+1)^2}\,\mathrm{d}\,x = \frac{\pi}{4}$
7. $\displaystyle\int_0^\infty \frac{1}{(x^2+1)(x^2+9)}\,\mathrm{d}\,x = \frac{\pi}{24}$
8. $\displaystyle\int_0^\infty \frac{x^2}{x^8+1}\,\mathrm{d}\,x = \frac{\pi}{8\sin\frac{3\pi}{8}}$

II[A] . Igazoljuk az alábbi egyenlőségeket.

1. $\displaystyle\int_0^\infty \frac{\cos x}{(x^2+1)(x^2+4)}\,\mathrm{d}\,x = \frac{\pi\,(2\,\mathrm{e}-1)}{12\,\mathrm{e}^2}$
2. $\displaystyle\int_0^\infty \frac{x\sin(2x)}{x^2+3}\,\mathrm{d}\,x = \frac{\pi}{2}\cdot\mathrm{e}^{-2\sqrt{3}}$
3. $\displaystyle\int_0^\infty \frac{x^2\sin x\cos x}{(x^2+1)^2}\,\mathrm{d}\,x = \frac{\pi}{4\,\mathrm{e}^2}$
4. $\displaystyle\int_0^\infty \frac{x^3\sin x}{(x^2+1)(x^2+4)}\,\mathrm{d}\,x = \frac{\pi\,(4-\mathrm{e})}{6\,\mathrm{e}^2}$
5. $\displaystyle\int_0^\infty \frac{\cos(2x)}{x^2+\frac{\pi^2}{4}}\,\mathrm{d}\,x = \mathrm{e}^{-\pi}$
6. $\displaystyle\int_0^\infty \frac{x\sin x}{(x^2+1)^2}\,\mathrm{d}\,x = \frac{\pi}{4\,\mathrm{e}}$
7. $\displaystyle\int_{-\infty}^\infty \frac{\cos x}{x^2-2x+2}\,\mathrm{d}\,x = \frac{\pi}{\mathrm{e}}\cos 1$
8. $\displaystyle\int_{-\infty}^\infty \frac{x\sin x}{x^2+2x+2}\,\mathrm{d}\,x = \frac{\pi(\sin 1+\cos 1)}{\mathrm{e}}$

III[A] . Igazoljuk az alábbi egyenlőségeket, ahol $\alpha\in\,]-1,1[$ paraméter.

1. $\displaystyle\int_0^\infty \frac{x^\alpha}{x^2+1}\,\mathrm{d}\,x = \frac{\pi}{2\cos\frac{\pi\alpha}{2}}$
2. $\displaystyle\int_0^\infty \frac{x^\alpha}{(x^2+4)^2}\,\mathrm{d}\,x = \frac{2^{\alpha-5}\pi(1-\alpha)}{\cos\frac{\pi\alpha}{2}}$
3. $\displaystyle\int_0^\infty \frac{x^\alpha}{x^3+1}\,\mathrm{d}\,x = \frac{\pi}{3\sin\frac{\pi(1+\alpha)}{3}}$
4. $\displaystyle\int_0^\infty \frac{x^{2+\alpha}}{(x^2+1)^2}\,\mathrm{d}\,x = \frac{\pi(\alpha+1)}{\cos\frac{\pi\alpha}{2}}$
5. $\displaystyle\int_0^\infty \frac{x^\alpha}{x^4+1}\,\mathrm{d}\,x = \frac{\pi}{4\sin\frac{\pi(1+\alpha)}{4}}$
6. $\displaystyle\int_0^\infty \frac{x^{4+\alpha}}{(x^2+1)^3}\,\mathrm{d}\,x = \frac{\pi(\alpha+1)(\alpha+3)}{16\cos\frac{\pi\alpha}{2}}$