A2 / ZH2 / A csoport

1. a) 2 db lin. fgtlen vektor 2 dimenziós teret feszít (1 pont)

b) $\begin{pmatrix} 1 & 1 \\ 2 & -1 \\ 0 & 1 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 1 \\ 5 \\ -1 \\ 2 \end{pmatrix}$ Keressük az $\begin{pmatrix} a \\ b \end{pmatrix}$ megoldást.

$$\left(\begin{array}{cc|c} 1 & 1 & 1 \\ 2 & -1 & 5 \\ 0 & 1 & -1 \\ 2 & 2 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -3 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

Tehát $a=2$, $b=-1$ a koordináták. (3 pont)

c) $\underline{v}_1 = \dfrac{\underline{u}_1}{\|\underline{u}_1\|} = \dfrac{1}{\sqrt{1+4+4}} \begin{pmatrix} 1 \\ 2 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 1/3 \\ 2/3 \\ 0 \\ 2/3 \end{pmatrix}$ (1 pont)

$$\tilde{\underline{v}}_2 = \underline{u}_2 - \langle \underline{u}_2, \underline{v}_1 \rangle \underline{v}_1 = \begin{pmatrix} 1 \\ -1 \\ 1 \\ 2 \end{pmatrix} - \underbrace{\left(\tfrac{1}{3} - \tfrac{2}{3} + 0 + \tfrac{4}{3}\right)}_{3/3} \begin{pmatrix} 1/3 \\ 2/3 \\ 0 \\ 2/3 \end{pmatrix} = \begin{pmatrix} 2/3 \\ -5/3 \\ 1 \\ 4/3 \end{pmatrix} = \begin{pmatrix} 2 \\ -5 \\ 3 \\ 4 \end{pmatrix} \frac{1}{3}$$

$$\underline{v}_2 = \frac{\tilde{\underline{v}}_2}{\|\tilde{\underline{v}}_2\|} = \frac{3\tilde{\underline{v}}_2}{\|3\tilde{\underline{v}}_2\|} = \frac{1}{\sqrt{54}} \begin{pmatrix} 2 \\ -5 \\ 3 \\ 4 \end{pmatrix}$$ ugyanis $\sqrt{4+25+9+16} = \sqrt{54}$.

(2 pont)

2. $$\begin{pmatrix} 3 & 6 & 9 & 30 \\ -1 & -2 & 2 & -7 \\ 2 & 4 & 8 & 26 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 3 & 6 & 9 & 30 \\ 2 & 4 & 8 & 26 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 4 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Tehát rang = 2 (3 pont)

3. Karakt. polinom: $\begin{vmatrix} 1-\lambda & 1 \\ 4 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 - 4 = \lambda^2 - 2\lambda - 3 = (\lambda+1)(\lambda-3)$

(3 pont)

$\lambda_1 = -1$ sajátértékhez: $\left(\begin{array}{cc|c} 2 & 1 & 0 \\ 4 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\underline{u}_1 = \begin{pmatrix} 1 \\ -2 \end{pmatrix}$ jó, hiszen $(2\ 1)\begin{pmatrix} 1 \\ -2 \end{pmatrix} = 0$.

$\lambda_2 = 3$ s.éhez: $\left(\begin{array}{cc|c} -2 & 1 & 0 \\ 4 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 2 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\underline{u}_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ jó. (3 pont)

4. $\dfrac{\partial}{\partial x}\left(e^{x^2+y} x^3 + \sin(x^2 y^3)\right) = e^{x^2+y} 2x^4 + e^{x^2+y} 3x^2 + \cos(x^2y^3)\, 2xy^3$ (2 pont)

$\dfrac{\partial}{\partial y}\left(\text{——} \right) = e^{x^2+y} x^3 + \cos(x^2y^3) \cdot x^2 3y^2$ (2 pont)