A2 / ZH2 / B csoport

① ⓐ 2db lin. ftlen vektor 2dimenziós teret feszít. (1pont)

ⓑ $\left(\begin{array}{cc|c} 1 & 2 & 1 \\ -2 & 1 & 8 \\ 3 & 1 & -7 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 5 & 10 \\ 0 & -5 & -10 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{array}\right)$

Tehát -3 és 2 a koordináták. (3pont)

ⓒ $\underline{v}_1 = \dfrac{\underline{u}_1}{\|\underline{u}_1\|} = \dfrac{1}{\sqrt{1+4+9}}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \dfrac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix}$ (1pont)

$\tilde{\underline{v}}_2 = \underline{u}_2 - \langle \underline{u}_2, \underline{v}_1 \rangle \underline{v}_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \dfrac{1}{\sqrt{14}}(2-2+3)\dfrac{1}{\sqrt{14}}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} - \dfrac{3}{14}\begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} = \dfrac{1}{14}\begin{pmatrix} 25 \\ 20 \\ 5 \end{pmatrix} = \dfrac{5}{14}\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$

$\underline{v}_2 = \dfrac{\tilde{\underline{v}}_2}{\|\tilde{\underline{v}}_2\|} = \dfrac{1}{\sqrt{25+16+1}}\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix} = \dfrac{1}{\sqrt{42}}\begin{pmatrix} 5 \\ 4 \\ 1 \end{pmatrix}$. (2pont)

② $\begin{pmatrix} 5 & 1 & 1 & 5 \\ 4 & -2 & -2 & 2 \\ 3 & -2 & -2 & 1 \end{pmatrix} \overset{\text{oszlop}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ -2 & 4 & 2 \\ -2 & 3 & 1 \end{pmatrix} \overset{\text{sor}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ 0 & 14 & 12 \\ 0 & 13 & 11 \end{pmatrix} \overset{\text{sor}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix}$

Tehát rang = 3 (3 pont)

③ Karakt. polinom: $\begin{vmatrix} 4-\lambda & -2 \\ 3 & -1-\lambda \end{vmatrix} = (\lambda-4)(\lambda+1)+6 = \lambda^2-3\lambda+2 = (\lambda-1)(\lambda-2)$

(3pont)

$\lambda_1 = 1$ sajátértékhez: $\left(\begin{array}{cc|c} 3 & -2 & 0 \\ 3 & -2 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 3 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\underline{u}_1 = \begin{pmatrix} +2 \\ 3 \end{pmatrix}$ jó sajátvektor

$\lambda_2 = 2$ s.értékhez: $\left(\begin{array}{cc|c} 2 & -2 & 0 \\ 3 & -3 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\underline{u}_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ jó. (3pont)

④ $f(x,y) = \dfrac{A(x,y)}{B(x,y)}$, $A(x,y) = \ln x \cdot \cos(y^2)$, $B(x,y) = \sin(x+y)$

$\dfrac{\partial}{\partial x} f(x,y) = \dfrac{A'_x(x,y)B(x,y) - A(x,y)B'_x(x,y)}{B(x,y)^2}$ (1pont), és $\dfrac{\partial}{\partial y} f(x,y)$ = hasonlóan

$\dfrac{\partial}{\partial x} A(x,y) = \dfrac{1}{x}\cos(y^2)$ (1pont) $\dfrac{\partial}{\partial x} B(x,y) = \cos(x+y) = \dfrac{\partial}{\partial y} B(x,y)$ (1pont)

$\dfrac{\partial}{\partial y} A(x,y) = \ln x(-\sin(y^2))2y$ (1pont)