A2 | ZH2 | C csoport

① ⓐ 2db lin. ftlen vektor 2dimenziós ~~teret~~ feszít. (1pont)

ⓑ
$$\left(\begin{array}{cc|c} 1 & 0 & 3 \\ -1 & 1 & 1 \\ -1 & -2 & -11 \\ 1 & -2 & -5 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & -2 & -8 \\ 0 & -2 & -8 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right)$$

tehát 3 és 4 a koordináták. (3pont)

ⓒ
$$\underline{v}_1 = \frac{\underline{u}_1}{\|\underline{u}_1\|} = \frac{1}{\sqrt{1+1+1+1}}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1/2 \\ -1/2 \\ -1/2 \\ 1/2 \end{pmatrix}$$ (1 pont)

$$\tilde{\underline{v}}_2 = \underline{u}_2 - \langle \underline{u}_2, \underline{v}_1 \rangle \underline{v}_1 = \begin{pmatrix} 0 \\ 1 \\ -2 \\ -2 \end{pmatrix} - \frac{1}{2}(0-1+2-2)\frac{1}{2}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ -2 \\ -2 \end{pmatrix} + \frac{1}{4}\begin{pmatrix} 1 \\ -1 \\ -1 \\ 1 \end{pmatrix}$$

$$\underline{v}_2 = \frac{\tilde{\underline{v}}_2}{\|\tilde{\underline{v}}_2\|} = \frac{1}{\sqrt{1+9+81+49}}\begin{pmatrix} 1 \\ 3 \\ -9 \\ -7 \end{pmatrix} = \frac{1}{\sqrt{140}}\begin{pmatrix} 1 \\ 3 \\ -9 \\ -7 \end{pmatrix} \qquad = \frac{1}{4}\begin{pmatrix} 1 \\ 3 \\ -9 \\ -7 \end{pmatrix}$$

(2 pont)

②
$$\begin{pmatrix} 5 & 1 & 1 & 5 \\ 4 & 2 & -2 & 2 \\ 3 & -2 & -7 & 1 \end{pmatrix} \overset{\text{oszlop}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ -2 & 4 & 2 \\ -2 & 3 & 1 \end{pmatrix} \overset{\text{sor}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ 0 & 14 & 12 \\ 0 & 13 & 11 \end{pmatrix} \overset{\text{sor}}{\sim} \begin{pmatrix} 1 & 5 & 5 \\ 0 & 1 & 1 \\ 0 & 0 & -2 \end{pmatrix}$$

tehát rang = 3

(3 pont)

③ Karakt. polinom:
$$\begin{vmatrix} 1-\lambda & -1 \\ 3 & 5-\lambda \end{vmatrix} = (\lambda-1)(\lambda-5)+3 = \lambda^2-6\lambda+8 = (\lambda-2)(\lambda-4)$$

$\lambda_1=2$ sajátértékhez $\left(\begin{array}{cc|c} -1 & -1 & 0 \\ 3 & 3 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad \underline{u}_1 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$ jó sajátvektor

$\lambda_2=4$ s.értékhez $\left(\begin{array}{cc|c} -3 & -1 & 0 \\ 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\quad \underline{u}_2 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ jó.

④
$$\frac{\partial}{\partial x}\left(\sin(x^3y^2)+\cos(y^2)\cos(x^2)\right) = \cos(x^3y^2)\,3x^2y^2 + \cos(y^2)(-\sin(x^2))\,2x$$

(2pont)

$$\frac{\partial}{\partial y}\left(\text{—''—}\right) = \cos(x^3y^2)\,x^3 2y - \cos(x^2)\sin(y^2)\,2y$$

(2pont)