A2 / ZH2 / D csoport

① ⓐ 2db lin.-ftlen vektor 2dimenziós teret feszít. (1pont)

ⓑ $\left(\begin{array}{cc|c} -1 & 1 & 1 \\ 2 & 1 & -5 \\ 2 & -3 & -1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 3 & -3 \\ 0 & -1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 0 & -2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$

tehát -2 és -1 a koordináták. (3 pont)

ⓒ $\underline{v}_1 = \frac{\underline{u}_1}{\|\underline{u}_1\|} = \frac{1}{\sqrt{1+4+4}} \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} -1/3 \\ 2/3 \\ 2/3 \end{pmatrix}$ (1pont)

$\tilde{\underline{v}}_2 = \underline{u}_2 - \langle \underline{u}_2, \underline{v}_1 \rangle \underline{v}_1 = \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} - \frac{1}{3}(-1+2-6)\frac{1}{3}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} =$

$= \begin{pmatrix} 1 \\ 1 \\ -3 \end{pmatrix} + \frac{5}{9}\begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix} = \frac{1}{9}\begin{pmatrix} 4 \\ 19 \\ -17 \end{pmatrix}$. $\underline{v}_2 = \frac{\tilde{\underline{v}}_2}{\|\tilde{\underline{v}}_2\|} = \frac{1}{\sqrt{4^2+19^2+17^2}} \begin{pmatrix} 4 \\ 19 \\ -17 \end{pmatrix}$

(2pont)

② $\begin{pmatrix} 3 & 6 & 9 & 30 \\ -1 & -2 & -2 & -7 \\ 2 & 4 & 8 & 26 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 3 & 6 & 9 & 30 \\ 2 & 4 & 8 & 26 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 0 & 0 & 3 & 9 \\ 0 & 0 & 4 & 12 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 2 & 7 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

tehát rang $= 2$ (3 pont)

③ karakt. polinom: $\begin{vmatrix} 1-\lambda & 2 \\ 3 & -4-\lambda \end{vmatrix} = (\lambda-1)(\lambda+4) - 6 = \lambda^2 + 3\lambda - 10 = (\lambda+5)(\lambda-2)$

$\lambda_1 = -5$ sajátértékhez: $\left(\begin{array}{cc|c} 6 & 2 & 0 \\ 3 & 1 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 3 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $\underline{u}_1 = \begin{pmatrix} 1 \\ -3 \end{pmatrix}$ jó sajátvektor
(3 pont) (3 pont)

$\lambda_2 = 2$ sajátértékhez: $\left(\begin{array}{cc|c} -1 & 2 & 0 \\ 3 & -6 & 0 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & -2 & 0 \\ 0 & 0 & 0 \end{array}\right)$ $u_2 = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ jó.

④ $\frac{\partial}{\partial x}\left(\cos(x^2y + xy^2) + e^{x+y}\ln(1+y+x)\right) = -\sin(x^2y+xy^2)(2xy+y^2) +$
$+ e^{x+y}\ln(1+y+x) + e^{x+y}\frac{1}{1+y+x}$

$\frac{\partial}{\partial y}\left(\text{—}\shortparallel\text{—}\right) = -\sin(x^2y+xy^2)(x^2+2xy) + e^{x+y}\ln(1+y+x) + e^{x+y}\frac{1}{1+y+x}$ (2+2 pont)