F2 pótZH I/A.

(1) $$\sum_{n=1}^{\infty} \frac{n}{n^2+1} \geq \sum_{n=1}^{\infty} \frac{n}{2n^2} = \frac{1}{2} \sum_{n=1}^{\infty} \frac{1}{n} = \infty$$

(3 pont) (2 pont)

Utolsó lépést tanultuk, pld integrál kritérium segítségével.

(2) $$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi} \int_{-\pi}^{0} -2\,dx + \frac{1}{2\pi} \int_0^{\pi} 2\,dx = 0.$$

$$a_k = \frac{1}{\pi} \int_{-\pi}^{0} -2\cos kx\,dx + \frac{1}{\pi} \int_0^{\pi} 2\cos kx\,dx = 0,$$ mert $f(x)$ páratlan, $\cos kx$ páros fgv.

(2pont)

$$b_k = \frac{1}{\pi} \int_{-\pi}^{0} -2\sin kx\,dx + \frac{1}{\pi} \int_0^{\pi} 2\sin kx\,dx = \frac{4}{\pi} \int_0^{\pi} \sin kx\,dx =$$

$$= \frac{4}{\pi} \left[-\frac{\cos kx}{k} \right]_0^{\pi} = \begin{cases} \frac{4}{\pi} \frac{1+1}{k}, & \text{ha } k \text{ páratlan} \\ 0, & \text{ha } k \text{ páros}. \end{cases}$$ (3pont)

Tehát $f(x) = \sum_{l=0}^{\infty} \frac{8}{\pi(2l+1)} \sin(2l+1)x$ a Fourier sor.

(1pont)

(3) $$\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 2 & 1 & 4 & 1 \\ 1 & 2 & a & b \end{array}\right) \underset{S_3-S_1}{\overset{S_2-2S_1}{\sim}} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & -5 & 0 & -15 \\ 0 & -1 & a-2 & b-8 \end{array}\right) \underset{S_3+S_2}{\overset{S_2/5}{\sim}} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & a-2 & b-5 \end{array}\right)$$ (2pont)

Tehát 0 megoldás: $a-2=0$, $b-5 \neq 0$ (1 pont)

∞ sok megoldás: $a-2=0$, $b-5=0$ (1 pont)

1 megoldás: $a-2 \neq 0$. (1pont)

(4) $$\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 2 & 3 & 4 & 0 & 1 & 0 \\ 1 & 3 & 6 & 0 & 0 & 1 \end{array}\right) \underset{S_3-S_1}{\overset{S_2-2S_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 2 & 5 & -1 & 0 & 1 \end{array}\right) \overset{S_3-2S_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -2 & 1 & 0 \\ 0 & 0 & 1 & 3 & -2 & 1 \end{array}\right) \underset{S_1-S_2-S_3}{\overset{S_2-2S_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 6 & -3 & 1 \\ 0 & 1 & 0 & -8 & 5 & -2 \\ 0 & 0 & 1 & 3 & -2 & 1 \end{array}\right)$$ (4p)

Avagy Cramer:

$\det A = (18+4+6) - (3+12+12) = 1$ (1pont)

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$ (0.5 pont)

$\det\begin{pmatrix} 3 & 4 \\ 3 & 6 \end{pmatrix} = 6$ stb.

$$\begin{matrix} 6 & 8 & 3 \\ 3 & 5 & 2 \\ 1 & 2 & 1 \end{matrix}$$ (2pont)

és (0.5p) transzpon:

$$\begin{pmatrix} 6 & -3 & 1 \\ -8 & 5 & -2 \\ 3 & -2 & 1 \end{pmatrix}$$