A2 pót ZH I/B

① $\sum_{n=1}^{\infty} \frac{n}{(n^2+1)^2} < \sum_{n=1}^{\infty} \frac{n}{n^4} = \sum_{n=1}^{\infty} \frac{1}{n^3} < \infty$. Utóbbi lépést akár pontosíthatod pl. integrálkritériummal!

(3p) (2p) és nemnegatív tagok. Konvergens.

② $a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)\,dx$. Mivel $f(x)$ páratlan, $\cos(kx)$, $k=0,1,2,\dots$ pedig páros, a szorzat páratlan, így $a_k = 0$ mind.

$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos kx\,dx$.

(2 pont)

$b_k = \frac{1}{\pi}\int_{-\pi}^{\pi/2} -2\sin kx\,dx + \frac{1}{\pi}\int_{\pi/2}^{\pi} 2\sin kx\,dx = \frac{4}{\pi}\int_{\pi/2}^{\pi} \sin kx\,dx =$

$$= \frac{4}{\pi}\left[\frac{-\cos kx}{k}\right]_{\pi/2}^{\pi} = \begin{cases} \frac{4}{\pi k}[-1+1] & \text{, ha } k \equiv 0 \pmod 4 \\ \frac{4}{\pi k}[1-0] & \text{, ha } k \equiv 1 \pmod 4 \\ \frac{4}{\pi k}[-1-1] & \text{ ha } k \equiv 2 \pmod 4 \\ \frac{4}{\pi k}[1-0] & \text{, ha } k \equiv 3 \pmod 4 \end{cases}$$

(3 pont)

Tehát $f(x) = \sum_{l=0}^{\infty} \frac{4}{\pi(2l+1)}\sin(2l+1)x - \sum_{l=0}^{\infty} \frac{8}{\pi(4l+2)}\sin(4l+2)x$ a Fourier sor.

(1 pont)

③ $\left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 2 & 1 & 4 & 1 \\ 4 & 2 & a & b \end{array}\right) \overset{\text{ld.}}{\underset{\text{A isop.}}{\sim}} \left(\begin{array}{ccc|c} 1 & 3 & 2 & 8 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & a-2 & b-5 \end{array}\right)$

(2 pont)

Tehát 0 megoldás: $a=2$, $b \neq 5$

∞ sok megoldás: $a=2$, $b=5$

1 megoldás: $a \neq 2$.

(1+1+1 pont)

④ $\left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 3 & 4 & 5 & 0 & 1 & 0 \\ 3 & 6 & 10 & 0 & 0 & 1 \end{array}\right) \underset{S_3-3S_1}{\overset{S_2-3S_1}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -3 & 1 & 0 \\ 0 & 3 & 7 & -3 & 0 & 1 \end{array}\right) \overset{S_3-3S_2}{\sim} \left(\begin{array}{ccc|ccc} 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & -3 & 1 & 0 \\ 0 & 0 & 1 & 6 & -3 & 1 \end{array}\right) \underset{S_1-S_2-S_3}{\overset{S_2-S_3}{\sim}} \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 10 & -4 & 1 \\ 0 & 1 & 0 & -15 & 7 & -2 \\ 0 & 0 & 1 & 6 & -3 & 1 \end{array}\right)$

(4 pont)

Avagy Cramer:

$\det A = (40+18+15) - (12+30+30) = 1$

(1 pont)

$\det\begin{pmatrix} 4 & 5 \\ 6 & 10 \end{pmatrix} = 10 \quad 15 \quad 6$

$\det\begin{pmatrix} 1 & 1 \\ 6 & 10 \end{pmatrix} = 4 \quad 7 \quad 3$

$\det\begin{pmatrix} 1 & 1 \\ 4 & 5 \end{pmatrix} = 1 \quad 2 \quad 1$

(2 pont)

$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$

$\begin{pmatrix} 10 & -4 & 1 \\ -15 & 7 & -2 \\ 6 & -3 & 1 \end{pmatrix}$

és transzponálás:

(0.5 + 0.5 pont)