1. Check whether the vector field $\overrightarrow{F}(x,y,z) = (6x+y\cos(xy), 2z+x\cos(xy), 2y)$ has a potential function or not! If there is, determine it!

2. Let H be a triangle with vertices $A = (0,0,1), B = (1,0,0)$ and $C = (0,1,0)$. Consider the constant vector field $\overrightarrow{F}(x,y,z) = (1,2,3)$. What is $\iint_H \overrightarrow{F} d\overrightarrow{A} =?$

3. Let $\overrightarrow{F}(x,y) = (xe^x - y^3, cos(y^2) + x^3)$ and let γ be a circle, centered at the origin with radius 1 and oriented anticlockwise. What is $\int_\gamma \overrightarrow{F} d\mathbf{r} =?$

4. Let $\overrightarrow{F}(x,y,z) = (3x^2y^2z, 2x^3yz, x^3y^2)$ and let γ be a curve with endpoints $A = (1,0,1)$ and $B = (2,1,-1)$. Determine $\int_\gamma \overrightarrow{F} d\mathbf{r} =?$

5. The surface S looks like a glass upside down. That is, there is the bottom of the glass
$$\{(x,y,z) : x^2 + y^2 \leq a^2 \text{ and } z = h\},$$
and the side of the glass
$$\{(x,y,z) : x^2 + y^2 = a^2 \text{ and } 0 \leq z \leq h\}.$$
Orientate the surface S with normal vector pointing out. Moreover, let $\overrightarrow{F}(x,y,z) = (z^2, x+2y+z, 3)$. By applying Gauss' Divergence Theorem, determine $\iint_S \overrightarrow{F} d\overrightarrow{A} =?$

6. We have the following informations about a vector field $\overrightarrow{F}$: at the point $P = (3,2,5)$, $\text{div}(\overrightarrow{F})(P) = -7$ and $\text{curl}(\overrightarrow{F})(P) = (1,0,1)$. Let S be the surface of the small ball centered at P with radius $r = 10^{-3}$, with orientation pointing outside. Give an estimate by using these informations for $\iint_S \overrightarrow{F} d\overrightarrow{A}$!

7. Let $\overrightarrow{F}(x,y,z) = (6x^2y - 4yz^3, 2x^3 - 4xz^3, -12xyz^2)$. Check that wether the vector field $\overrightarrow{F}$ is conservative or not! If it is, provide the potential function!

8. Let γ be a curve with parametrization $\mathbf{r}(t) = (t, t^2, t^3)$, $0 \le t \le 1$. Moreover, given the vector field $\overrightarrow{F}(x,y,z) = (-x^3 + z, \frac{z-y}{x}, xy)$. Calculate $\int_\gamma \overrightarrow{F} d\mathbf{r}$.

9. Let $\mathcal{F}$ be the triangle with vertices $A = (1,0,0), B = (0,1,3)$ and $C = (0,0,1)$. Orientate the surface $\mathcal{F}$ upwards. Let $\overrightarrow{F}(x,y,z) = (y, z, x^2)$. What is $\iint_{\mathcal{F}} \overrightarrow{F} d\overrightarrow{A}$?

10. Using Stokes' Theorem, calculate $\int_\gamma \overrightarrow{F} d\mathbf{r}$, where $\overrightarrow{F}(x,y,z) = (2xy, x^2, z^2)$ and γ is the intersection of the paraboloid $x^2 + y^2 = z$ and the plane $z = y$, oriented anticlockwise.

11. Let $\overrightarrow{F}(x,y,z) = (x,y,z)$ and let $\mathcal{F} = \{(x,y,z) : x^2 + z^2 = 1 \text{ and } -2 \le y \le 1\}$ with normal vector pointing out. What is $\iint_{\mathcal{F}} \overrightarrow{F} d\overrightarrow{A} =$?

12. Using Gauss' Theorem, determine $\iint_{\mathcal{F}} \overrightarrow{F} d\overrightarrow{A}$, where $\overrightarrow{F}(x,y,z) = (x^2 + e^{yz}, y - z\arctan(x), x^2y^3)$ and $\mathcal{F} = \{(x,y,z) : x^2 + y^2 = 4 \text{ and } 0 \le z \le 3\}$ with normal vector pointing out.

13. Using Green's Theorem, calculate the area of the domain which is surrounded by the curve $\mathbf{r}(t) = (\cos^3(t), \sin^3(t))$, $0 \le t \le 2\pi$.

14. Let γ be the straight line connecting $A = (1,1,1)$ and $B = (1,2,4)$, and let $\overrightarrow{F}(x,y,z) = (2xyz, x^2z, x^2y)$. What is $\int_\gamma \overrightarrow{F} d\mathbf{r} =$?

15. Using the planar version of the Curl-test, provide that the following vector fields are conservative or not. If it is then give the potential function. Give

reasoning if the Curl-test is inconclusive!

(a) $\overrightarrow{F}(x,y) = (-y, x)$, **(b)** $\overrightarrow{F}(\mathbf{r}) = \|\mathbf{r}\| \cdot \mathbf{r}$,

(c) $\overrightarrow{F}(x,y) = \left(\frac{y}{x^2+y^2}, \frac{-x}{x^2+y^2}\right)$, **(d)** $\overrightarrow{F}(x,y) = \left(\frac{y}{\sqrt{x^2+y^2}}, \frac{-x}{\sqrt{x^2+y^2}}\right)$.

16. Let $\overrightarrow{F}(x,y,z) = (x^{-2}, z, y)$. Is it possible to use the Curl-test? Try to guess the potential function! Is the vector field $\overrightarrow{F}$ conservative?
17. Let $\mathcal{S} = \{(x,y,z) : x^2 + y^2 = 4 \text{ and } 0 \le z \le 2\}$ be cylindrical surface with normal vector pointing in the direction of the axes z. Let $\overrightarrow{F}(x,y,z) = (y+1, x, z^2)$. What is $\iint_{\mathcal{S}} \overrightarrow{F} d\overrightarrow{A} =$?
18. Let $\overrightarrow{F}(\mathbf{r}) = \frac{1}{\|\mathbf{r}\|^3}\mathbf{r}$ and let $\mathcal{F}$ the surface of the unit ball with normal vector pointing out. What is $\iint_{\mathcal{F}} \overrightarrow{F} d\overrightarrow{A} =$?