Probability 1 - Practice

Week 1 2018.09.04.

1.1 John, Jim, Jay and Jack have formed a band consisting of 4 instruments.

(a) If each boys can play all 4 instruments, how many different arrangements are possible?

(b) What if John and Jim can play all four instruments, but Jay and Jack can only play piano and drums?

1.2 For years, telephone area codes in the United States and Canada consisted of a sequence of three digits. The first digit was an integer between 2 and 9, the second digit was either 0 or 1, and the third digit was any integer from 1 to 9.

(a) How many area codes were possible?

(b) How many area codes starting with 4 were possible?

1.3 In how many ways can 8 people be seated in a row if

(a) there are no restrictions on the seating arrangement?

(b) persons A and B must sit next to each other?

(c) there are 4 men and 4 women, and no 2 men or 2 women can sit next to each other?

(d) there are 5 men and they must sit next to each other?

(e) there are 4 married couples, and each couple must sit together?

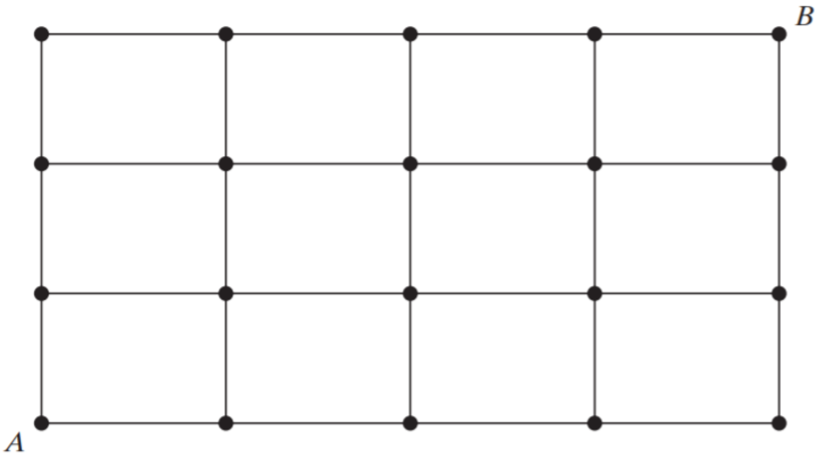
1.4 Consider a group of 20 people. If everyone shakes hands with everyone else, how many handshakes take place?

1.5 How many 5-card poker hands are there?

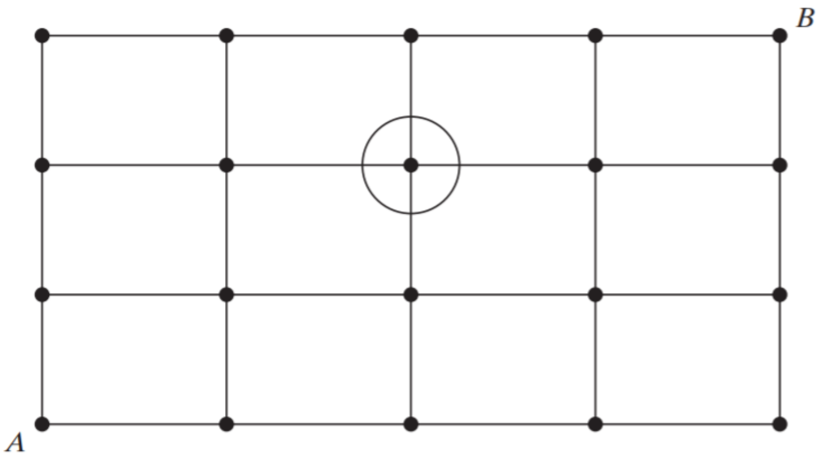
1.6 A dance class consists of 22 students, of which 10 are women, and 12 are men. If 5 men and 5 women are to be chosen and then paired off, how many results are possible?

1.7 A commitee of 7, consisting of 2 Republicans, 2 Democrats, and 3 Independents, is to be chosen from a group of 5 Republicans, 6 Democrats, and 4 Independents. How many commitees are possible?

1.8 Consider the grid of points shown here. Suppose that, starting at the point labeled A, you can go one step up, or one step to the right at each move. This procedure is continued until the point labeled B is reached. How many different paths from A to B are possible?



HW 1.9 In the previous problem, how many different paths are there from A to B that go through the point circled in the following lattice?



HW 1.10 If 8 identical blackboards are to be divided among 2 schools, how many divisions are possible? How many if each school must receive at least 1 blackboard?

1.11 We put n (distinguishable) balls randomly into n (distinguishable) urns. What is the probability, that exactly one urn remains empty?

1.12 We put 8 rooks (tower) randomly onto a chess board. What is the probability that non of the rooks can knock each other out?

1.13 From a population of n balls in a bucket, we color N to be red, and the rest of them to be blue. Now, let's chose a sample of size r from the population. What is the probability that none of the selected balls are red? If:

(a) We don't put them back into the bucket after each draw?

(b) We put them back into the bucket after each draw?

♣ **1.14** There are n pairs of shoes in a closet. We chose $2r$ shoes randomly ($2r \leq n$). What is the probability, that:

(a) There are no pairs chosen?

(b) There is exactly one pair chosen?

(c) There are exactly two pairs chosen?

1.15 Prove that:

$$\binom{n+m}{r} = \binom{n}{0}\binom{m}{r} + \binom{n}{1}\binom{m}{r-1} + \cdots + \binom{n}{r}\binom{m}{0}$$

1.16 Using the previous exercise, prove that:

$$\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$$

HW 1.17 Consider the following combinatorial identity

$$\sum_{k=1}^{n} k \cdot \binom{n}{k} = n \cdot 2^{n-1}$$

Present a combinatorial argument for this identity by considering a set of n people and determining in two ways, the number of possible selections of a commitee of any size, and a chairperson for the committee.