Probability 1 - Practice

Week 2 2018.09.11.

2.1 Let A, B and C be three events. Use set theory operations to express the following events:

(a) Out of A, B and C, exactly k events occurs ($k = 0, 1, 2$)

(b) Out of A, B and C, at least k events occurs ($k = 1, 2$)

2.2 Let's prove the following equalities

(a) $A \cap B \setminus C = (A \setminus C) \cap (A \setminus B)$

(b) $A \circ (B \circ C) = (A \circ B) \circ C$

HW (c) $A \circ C \subset (A \circ B) \cup (B \circ C)$

2.3 How many different outcomes can the experiment have? Identify the sample space of the experiment:

(a) We throw 3 different coins and 2 identical dice.

(b) We throw 3 identical black dice, and 2 identical white dice.

2.4 What has a higher probability? Throwing a die four times, and rolling 6 at least once, or throwing two dice 24 times, and rolling 6 on both together at least once.

2.5 Adam and Eve play ping-pong, and let's assume that they are both equally good at the game. Let A, B be the following events:

$$A := \{\text{Adam wins exactly 3 matches out of 4}\}$$
$$B := \{\text{Eve wins exactly 8 matches out of 5 }\}$$

Which event has a higher probability? First guess, then calculate the exact probabilities.

2.6 We throw a coin until it lands on the same side twice in a row. Prove that the probability of every n long sequence is 2^{-n}. Write down the probability space for the experiment. What is the probability of the following events?

$$A := \{\text{The experiment ends after less than 6 throws}\}$$
$$B := \{\text{The experiment ends after an even amount of throws }\}$$

2.7 Show that for any A, B events:

$$-\frac{1}{4} \leq P(A \cap B) - P(A)P(B) \leq \frac{1}{4}$$

2.8 (a) Let A and B be two events. If $P(A) \geq 0.8$ and $P(B) \geq 0.5$ then prove the following: $P(A \cap B) \geq 0.3$

(b) Let's prove the following equality for any $A_1, A_2, \ldots, A_n$ events:

$$P(A_1 \cap A_2 \cap \cdots \cap A_n) \geq P(A_1) + P(A_2) + \cdots + P(A_n) - (n-1)$$

HW 2.9 (a) For any A, B and C, prove that:

$$P(A \circ C) \leq P(A \circ B) + P(B \circ C)$$

(b) Prove that if $P(A \circ B) = 0$, then $P(A) = P(B)$.

2.10 (a) We throw a die 6 times. What is the probability, that $1, 2, 3, 4, 5$ and 6 all get thrown?

(b) We throw a dice 10 times. What is the probability that $1, 2, 3, 4, 5$ and 6 all get thrown at least once?

2.11 Using the inclusion-exclusion identity, show that:

$$\sum_{k=0}^{n} (-1)^k \binom{n}{k} = 0$$

2.12 There are 6 red, 6 white, and 7 blue balls in an urn. We draw 5 without repetition. What is the probability that we draw at least one of each color?

HW 2.13 An urn contains 5 red, 6 blue, and 8 green balls. What is the sample space, if a set of 3 balls is randomly selected without repetition? What is the probability that each of the balls will be:

(a) of the same color?

(b) of different colors?

HW 2.14 We throw 3 dice in the air. Dice that are the same color are indistinguishable. How many different outcomes can the experiment have, if:

(a) the dice are all the same color?

(b) two dice are black, and the third is white?

(c) all three are different colors?