Probability Theory 2.

Examination topics

1. **Convolution I.** Definition of Stielties integral, basic properties, definition of convolution (discrete and continuous case), basic properties, smoothening theorems, examples: BIN*BIN, POI*POI, GEO*GEO, UNI*UNI, GAU*GAU, EXP*EXP.

2. **Convolution II.** Gamma-distribution, connection with the Poisson- and Exponential distributions, Poisson process, CLT for exponential distribution, Euler's Gamma-functions, basic properties Chi^2 distribution.

3. **Prob. generating function I.** Definition of probability generating function, basic properties, examples (BIN, POI, GEO). Moments, reconstruction of the distribution from the generator function. Convolution, mixed distribution, prob. generating function of randomly many terms, branching (Galton-Watson) processes, probability of extinction.

4. **Prob. generating function II.** Simple random walk on **Z**, reaching times, recurrence, transience.

5. **Prob. generating function III.** Weak convergence of sequences of **N** valued random variables and the pointwise convergence of the prob. generating function, tightness, POI approximation with BIN.

6. **Types of convergences** Almost sure (strong) convergence, Convergence in probability (Stochastic conv.), convergence in L^1; L^2; L^p. Relations between them. (strong => in prob., in prob. => strong on subsequence, L^1 ⇔ in prob. + uniformly integrability, Scheffé's Theorem, counterexamples)

7. **Weak Law of Large Numbers I.** Markov's and Chebisev's inequalities, weak law of large numbers (with second moment), Bernstein approximation, Coupon-collector problem.

8. **Weak Law of Large Numbers II.** truncation, weak law of large numbers (with first moment) St. Petersburg paradox.

9. **Measure theory I.** σ-algebra, definition of measure, basic properties, generated σ-algebra, measurable function, definition of probability space, definition of integral, basic properties.

10. **Measure theory II.** Fubini's theorem, bounded convergence theorem, Fatou's lemma, monoton convergence theorem, dominated convergence theorem.

11. **Strong Law of Large Numbers I.** Borel-Cantelli lemmas. Strong law of large numbers with fourth moment, Kolmogorov's inequality. $\sum \mathbf{D}^2(X_k)<\infty \Rightarrow \sum(X_k-\mathbf{E}(X_k))$ exists almost surely,

12. **Strong Law of Large Numbers II.** Strong law of large numbers with first moment, tail σ-algebra, Kolmogorov 0-1 law, applications.

13. **Large deviations.** Logarithmic moment generating function, basic properties, rate function (Legendre transform), properties, Cramér's Large deviation theorem with first moment.

14. **Characteristic function I.** Definition, basic properties (bounds, uniform continuity, positive definite), connection with lattice distributed random variables, Bochner's Theorem (without proof), characteristic function of notable distributiosn (EXP, UNI, normal). Cauchy distribution and its characteristic function.

15. **Characteristic function II.** Moments and the derivatives of the characteristic function, smoothness of distribution function and the decay of the characteristic function at $\pm\infty$, characteristic function of sum of independent random variables, reconstruction of the distribution from the characteristic function.

16. **Weak convergence of distributions I.** Definition of weak convergence of probability distributions (with probability measures), Portmanteau's theorem (equivalences). Weak convergence of distribution and pointwise convergence of distribution functions, Poisson approximation, DeMoivre's theorem.

17. **Weak convergence of distributions II.** Tightness and Prohorov's theorem, Lévy's lemma and continuity theorem, central limit theorem.