COVERING PROPERTIES OF IDEALS

MAREK BALCERZAK, BARNABÁS FARKAS, AND SZYMON GŁĄB

Abstract. M. Elekes proved that any infinite-fold cover of a σ-finite measure space by a sequence of measurable sets has a subsequence with the same property such that the set of indices of this subsequence has density zero. Thanks to this theorem he gave a new proof for the random-indestructibility of the density zero ideal. He asked about other variants of this theorem.

We present some negative results and discuss the category case when the set of indices of the required subsequence should be in a fixed ideal $\mathcal{J}$ on ω. We introduce the notion of the $\mathcal{J}$-covering property of a pair $(\mathcal{A}, I)$ where $\mathcal{A}$ is a σ-algebra on X and $I \subseteq \mathcal{P}(X)$ is an ideal. We investigate connections between this property and forcing-indestructibility of ideals. Also, we study the $\mathcal{J}$-covering property for the pairs $(\mathrm{Borel}(X\times Y), I\otimes K)$ where X, Y are Polish spaces, and $(\mathcal{P}(\omega), \mathcal{I})$ where $\mathcal{I}$ is an ideal on ω.

Institute of Mathematics, Technical University of Łódź, ul. Wólczańska 215, 93-005 Łódź, Poland

E-mail address: marek.balcerzak@p.lodz.pl

Institute of Mathematics, Budapest University of Technology and Economics, Hungary

E-mail address: barnabasfarkas@gmail.com

Institute of Mathematics, Technical University of Łódź, ul. Wólczańska 215, 93-005 Łódź, Poland

E-mail address: szymon.glab@p.lodz.pl

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