Sample Final Exam
Mathematics A1a

1. Given the equation of the line $e \; : \dfrac{x-4}{2} = \dfrac{2-y}{2} = z+1$ and the point $P(8,4;2)$, find

a.) the equation of the line through P and parallel to e,
b.) the equation of the plane through P and perpendicular to e,
c.) the equation of the plane made by the line e and the point P. (10 points)

2. Find in algebraic form: $\sqrt{i}$. (4 points)

2. a.) When do we say that the function $f(x)$ is continuous at the point $x = x_0$? Put down the definition.
b.) Which value of a (if any) makes the following function continuous at $x = 0$?

$$f(x) = \begin{cases} \dfrac{\sinh^2 x}{x^3 - x^2}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$ (6 points)

3. Based on the definition of the derivative show that $(\sin x)' = -\cos x$. (6 points)

4. a.) Give the Taylor-polynomial of degree 2 generated by the function $f(x) = \sin^2 x$ at the point $x = 0$.
b.) Use this polinomial to approximate the value of $\sin^2(0.1)$.
c.) Estimate the error of this approximation. (12 points)

5. True or false? Give reason for your answer:
a.) If the sequence $\{a_n\}$ tends to plus infinity then it is monotonically increasing.
b.) If the sequence $\{a_n\}$ is monotonically increasing then it tends to plus infinity.
c.) The function $f(x) = x \sin 2x$ is odd.
d.) If [redacted], when [redacted] and [redacted], when [redacted] then the function $f(x)$ has a point of inflection at [redacted].
(8 points)

6. Sketch te graph of the function $y = \dfrac{1}{1-x^2}$. (Find the domain, name any relative extrema, points of inflection, limits at $\pm\infty$, describe monotonity, concavity, give the range.)
(14 points)

7. a.) $\displaystyle\int \frac{\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^2}+3}\,dx = ?$ (Hint: $u = \sqrt[3]{x-1}$.) b.) $\displaystyle\int \frac{1}{x^2+3x-4}\,dx = ?$

c.) $\displaystyle\int_2^\infty \frac{1}{x^2+3x-4}\,dx = ?$ d.) $\displaystyle\int_{-\pi/2}^{\pi/2} x \cdot \cos^2 x dx = ?$ e.) $\displaystyle\int_0^4 \frac{1}{\sqrt{4x+9}}\,dx =$ (20 points)

8. Find the area of the region enclosed by the curves $y = \dfrac{3}{2+x^2}$, $y = x^2$. (10 points)

Passing limit: Total score: 90 points

- at least 12 points on problem 7 and 8,
- at least 36 points total