Excercises
Matematics A1a
Application of differentiation

1. Give the derivatives of these implicitly given functions.

a.) $x^3 + y^3 - 3xy = 0$, b.) $y = \frac{x}{x+y}$ c.) $y = \sin(x+y)$

2. Find an equation for the tangent to the ellipse $x^2 + 2y^2 = 3$ at the point (1;1).

3. Give the equations of the lines, that touch the hyperbola $x^2 - 4y^2 = 4$ and are parallel to the line $x - 5y + 2 = 0$.

4. Find the equations of the tangents and normals of the following curves at the given points :

a.) $\begin{cases} x = 3\cos^3 t \\ y = 3\sin^3 t \end{cases}$ $t_0 = \frac{\pi}{3}$, b.) $\begin{cases} x = \text{ch}t \\ y = \text{sh}t \end{cases}$ $t_0 = \ln 3$.

5. Give the equations of the lines, that touch the parametrically given ellipse $x = 2\cos t,\ y = 4\sin t$ and go through the point (3;0) .

6. The perimeter of a sector is 4,8 m. What value of the radius will maximize the area of the sector?

7. A trapezoid lying on the diameter is drawn in a semicircle with a radius of $r = 4$. What length of the sides of the trapezoid will maximize the area of it?

8. Give the height and the radius of the cylinder inscribed into a given cone to maximize the volume of the cylinder.

9. Give the height and the radius of the cone iscribed into a sphere of radius 1 to maximize the volume of the cone.

10. Which point of the parabola $y = \frac{x^2}{8}$ is at the least distance from the point (0;6)?

11. Standing on a horizontal field we throw a stone to a direction, whose angle to the horizontal equals to α. Which value of α will maximize the distance, where the stone falls on the ground?

12.* Two men carry a ladder in horizontal position. How long can the ladder at the longest be to allow the men to turn in a passage bend? The angle of the passages is 90°, the width of the passages are 3 m and 5 m.

13. Give the curvature of the following functions in the given points:

a.) $y = x^3 - 1,\ x_0 = 1$, b.) $y = \ln x^2,\ x_0 = 1$, c.) $y = \frac{1}{x},\ x_0 = 2$.

14. Give the largest curvature of the curve $y = \ln x$.

15. Give the Taylor polynomial of order 4 generated by the following functions at $x_0 = 0$:

a.) $y = \sqrt[3]{e^x}$, b.) $y = \cos(x^2)$, c.) $y = \text{ch}^2 x$, d.) $y = x \sin 2x$.

16. Give the Taylor polynomial of order 3 generated by the function $y = \frac{x+3}{x}$ at $x_0 = 1$.

17. Calculate the approximate value of these functions with the help of the Taylor polynomial of order 3, and estimate the error:

a.) $\sin 0{,}4$ b.) sh 0,3.

18. Calculate the approximate value of e with the help of the Taylor polynomial of order 3, and estimate the error.

19. Use the Taylor polynomial of order 3 by $y = \sqrt{x}$ at $x_0 = 4$ to calculate the approximate value of $\sqrt{6}$.

20. Calculate ln 1,1 with an error of less than 10^{-5}.

INDEFINITE INTEGRALS:

1. $\int \frac{x^3-6x+5}{x}dx$,
2. $\int \frac{\sqrt[3]{x^2}-\sqrt[4]{x}}{x}dx$
3. $\int \frac{2\cdot 5^x-5\cdot 2^x}{5^x}dx$
4. $\int (2x+5)^{3/2}dx$
5. $\int \sinh(5x-7)dx$
6. $\int \frac{dx}{\cos^2(4-3x)}$
7. $\int \frac{dx}{4-3x}$
8. $\int \frac{e^{7x+1}}{e^{2x}}dx$
9. $\int \sqrt{5-2x}dx$
10. $\int \frac{x^2}{3+x^3}dx$
11. $\int \frac{dx}{x\ln x}$
12. $\int \cot 2x dx$
13. $\int \frac{2x+3}{x^2+3x}dx$
14. $\int \frac{e^{3x}}{5+12e^{3x}}dx$
15. $\int \frac{\sin x}{1-\cos x}dx$
16. $\int \sin^3 x\cdot\cos x dx$
17. $\int x\sqrt{3-x^2}dx$
18. $\int \frac{(\arctan x)^2}{1+x^2}dx$
19. $\int \frac{\sqrt[3]{\tan x}}{\cos^2 x}dx$
20. $\int \frac{3x-1}{x^2-2x+10}dx$
21. $\int \frac{3x-1}{x^2-2x-3}dx$
22. $\int \frac{dx}{\sqrt{x^2+x+1}}$
23. $\int \frac{dx}{\sqrt{4x^2+7x}}$
24. $\int \frac{x-1}{\sqrt{3-2x-x^2}}dx$
25. $\int \frac{3x+2}{\sqrt{x^2-6x+25}}dx$
26. $\int \frac{\cot x}{\sqrt[10]{\ln\sin x}}dx$
27. $\int \frac{1+2x^2}{x^2(1+x^2)}dx$
28. $\int \tan^2 x dx$
29. $\int x\cdot e^{-x}dx$
30. $\int (x-2)\cos 3x dx$
31. $\int (x^2-1)\sinh(-x)dx$
32. $\int e^{5x}\cos 2x dx$
33. $\int x^2\ln x dx$
34. $\int \arccos\frac{x}{2}dx$
35. $\int (\sinh x+\cosh x)e^{-x}dx$
36. $\int \frac{2^{3x}}{1+2^{3x}}dx$
37. $\int \sqrt{3-2x-x^2}dx$
38. $\int \sqrt{5-2x+x^2}dx$
39. $\int 3x\sqrt{x^2-1}dx$
40. $\int \sqrt{2x^2-8}dx$
41. $\int \frac{e^x}{e^{-x}+3}dx$
42. $\int \frac{x^2-4x+7}{x-2}dx$
43. $\int \frac{4}{e^{2x}-4}dx$
44. $\int \arctan\sqrt{x}dx$
45. $\int e^{\sqrt{x}}dx$
46. $\int x\sin x\cos x dx$
47. $\int \cos^2 2x dx$
48. $\int \sin^2 x\cos^2 x dx$
49. $\int \sin^2 x\cos x dx$
50. $\int x\sin^2 x dx$
51. $\int e^{\sin x}\cos x dx$
52. $\int \frac{dx}{\sin x}$
53. $\int \sin 3x\sin 8x dx$
54. $\int \cos x\cos 5x dx$
55. $\int \cos^3 x dx$
56. $\int \cos^4\frac{x}{2}dx$
57. $\int \cos^3 x\sin^2 x dx$
58. $\int e^x\cos^2 x dx$
59. $\int \frac{dx}{\sinh x+\cosh x}dx$
60. $\int \sqrt[3]{\cos^7(3x+\pi)}\sin(3x+\pi)dx$

61. Give the following definite integrals:

a.) $\int_0^{\pi} \sin 2x \cos 2x dx$, b.) $\int_1^2 \frac{\sqrt{\ln x}}{x} dx$ c.) $\int_{-\pi/4}^{\pi/4} x \sin x dx$, d.) $\int_1^e \ln^2 x dx$.

62. Find the area bounded by these areas:

a.) x-axis, line $x = 2$, curve $y = \sqrt{x}$;

b.) $y = e^x$, $y = e^{-x}$, $x = 1$;

c.) $y = x^2$, $y = \frac{x^2}{2}$, $y = 2x$

d.) $y = \ln 2x$, its tangent, that goes through the origin, x-axis.

63. The region in the a.) part of the previous exercise is revolved about the x-axis. Give the surface and volume of the generated solid.

64. The parametrical equations of an astrois are $x = a\sin^3 t$, $y = a\cos^3 t$, $0 \le t \le 2\pi$. Give the area of the curve, the length of it, the surface and the volume of the solid, generated by revolving the curve about the x-axis.

65. Számítsa ki az $y = \sin\frac{x}{2}$ görbe első pozitív félhulláma alatti területet, súlypontjának koordinátáit és az x tengely körüli forgatásával nyert forgástest térfogatát!

66. Számítsa ki az $y = \frac{2}{3}(x-8)^{3/2}$ függvény görbéjének ívhosszát a [8;16] intervallumon.

67. Számítsa ki az alábbi görbék által határolt síkrészek területét és adja súlypontjaik helyét:

a.) $y = \sqrt[3]{x}, y = 0, x = 8$, b.) $y = x^2, y = 9$, $x = 0$, $x \ge 0$

68. Számítsa ki az alábbi improprius integrálokat:

a.) $\int_2^{\infty} \frac{1}{x^3} dx$, b.) $\int_{-\infty}^{\infty} \frac{3}{4x^2+1} dx$, c.) $\int_0^{\infty} xe^{-2x} dx$, d.) $\int_{-2}^{2} \frac{dx}{\sqrt{4-x^2}} dx$.