

Exercises
Mathematics A1
Numerical sequences

1. Let $a_n = \frac{2n-1}{5n+2}$. Find the smallest positive integer N such that for every $\forall n \geq N$ the difference between a_n and the limit of $\{a_n\}$ be less than $\varepsilon = 0,01$.

2. Write the first 5 terms of the following sequences. Check if the sequence is bounded, monotonic, convergent. Find the limit of the sequence (if there is any).

$$\text{a.) } a_n = n^2 + (-1)^n n^2, \quad \text{b.) } a_n = \frac{n-1}{n+1}, \quad \text{c.) } a_n = \frac{5^n}{n!}.$$

3. Convergent or divergent? If converges, find its limit.

$$\text{a.) } a_n = \frac{(3n-1)(n+2)}{(1-n)(2n+5)}, \quad \text{b.) } a_n = \frac{4^n}{3 \cdot 4^n + 2}, \quad \text{c.) } a_n = n \left(\sqrt{n^2 + 1} - n \right),$$

$$\text{d.) } a_n = \frac{2+5+8+\dots+(3n-1)}{n}, \quad \text{e.) } a_n = \left(\frac{n-2}{n} \right)^{3n+1}, \quad \text{f.) } a_n = \left(1 + \frac{1}{\sqrt{n}} \right)^n.$$

$$\text{g.) } a_n = \frac{\sin(2n)}{n^3}, \quad \text{h.) } a_n = \sqrt{n^2 + 1} - \sqrt{n^2 - 1}, \quad \text{i.) } a_n = \frac{n - \sqrt[3]{n^2}}{n + \sqrt{n^2 + 1}}$$

$$\text{j.) } a_n = \frac{(-1)^n + 5 \cdot 4^n - 6^n}{6^{n+2}}, \quad \text{k.) } a_n = \sqrt{\frac{n^3 + (-1)^n n^3}{3n^3 + n + 8}}$$