Sample Final Exam
Mathematics A1a

1. Given the equation of the line $e : \frac{x-4}{2} = \frac{2-y}{2} = z+1$ and the point $P(8,4;2)$, find

a.) the equation of the line through P and parallel to e,
b.) the equation of the plane through P and perpendicular to e,
c.) the equation of the plane made by the line e and the point P. (7 points)

2. Find in algebraic form: $\sqrt{i}$. (2 points)

3. a.) When do we say that the function $f(x)$ is continuous at the point $x = x_0$? Put down the definition.

b.) Which value of a (if any) makes the following function continuous at $x = 0$?

$$f(x) = \begin{cases} \frac{\sinh^2 x}{x^3 - x^2}, & \text{if } x \neq 0 \\ a, & \text{if } x = 0 \end{cases}$$ (5 points)

4. Based on the definition of the derivative show that $(\sin x)' = -\cos x$. (4 points)

5. Give the Taylor-polynomial of degree 5 generated by the function $f(x) = x \sinh 2x$ at the point $x = 0$. (4 points)

6. True or false? Give reason for your answer:

a.) If the sequence $\{a_n\}$ tends to plus infinity then it is monotonically increasing.

b.) If the sequence $\{a_n\}$ is monotonically increasing then it tends to plus infinity.

c.) The function $f(x) = x \sin 2x$ is odd.

d.) If $f''(x) < 0$, when $x < 0$ and $f''(x) > 0$, when $x > 0$ then the function $f(x)$ has a point of inflection at $x = 0$. (8 points)

7. Sketch te graph of the function $y = \frac{1}{1-x^2}$. (Find the domain, name any relative extrema, points of inflection, limits at $\pm\infty$, describe monotonity, concavity, give the range.) (10 points)

8. a.) $\int \frac{\sqrt[3]{x-1}}{\sqrt[3]{(x-1)^2}+3} dx = ?$ (Hint: $u = \sqrt[3]{x-1}$.) b.) $\int \frac{1}{x^2+3x-4} dx = ?$

c.) $\int_2^{\infty} \frac{1}{x^2+3x-4} dx = ?$ d.) $\int_{-\pi/2}^{\pi/2} x \cdot \cos^2 x dx = ?$ (12 points)

9. Converges or diverges? Give reason for your answer. $\int_{2\pi}^{\infty} \frac{3-\cos^2 x}{x\sqrt{x}} dx$ (3 points)

10. Find the area of the region enclosed by the curves $y = \frac{3}{2+x^2}$, $y = x^2$. (5 points)

Passing limit: Total score: 60 points

- at least 8 points on problem 8, 9 and 10,
- at least 24 points total