

SAMPLE TEST # 2
Mathematics A1
Dec 2009

1. Answer if the following statements are true or false. Give reason for your answer.

a.) If $f(x)$ is increasing monotonically in the interval $(a;b)$ then $f(x)$ is differentiable in $(a;b)$.

b.) If $\int f(x)dx = F(x) + C$ then $\int \frac{1}{f(x)} dx = \frac{1}{F(x)} + C$. (2 points)

2. Find the derivative of the functions:

a.) $y = x^2 \left(1 - \frac{3}{\tan x}\right)^3$ b.) $y = \ln \sqrt{\frac{x^3}{x-1}}$ (2 points)

3. Inscribe a rectangle into the region in the first quadrant bounded by the x-axis, y-axis and the curve $y = 4 - x^2$. Let the sides of the rectangle be parallel to the axis. Find the dimensions of the rectangle with largest area. (3 points)

4. Given the function $y = \frac{x-10}{(x-4)^2}$, find the domain, name any relative extrema, points of inflection, limits at $\pm\infty$ and at $x = 4$, describe monotonicity, concavity, give the range. Sketch the graph of the function. (4 points)

5. Give the Taylor's polynomial of order 4 generated by the function $y = x + \frac{1}{e^{(x^2)}}$ at the point $x = 0$. (2 points)

6. Find the equation of the line tangent to the curve $x = t^2$, $y = \sqrt{1+t^4}$ at the point $t = 1$. (2 points)

7. a.) $\int \left(x - \frac{1}{\sqrt{x}}\right)^2 dx + \int \frac{\ln^2 x}{x} dx$, b.) $\int \frac{1}{x^2 + 2x - 8} dx$ (3 points)

8. Find the area of the region enclosed by the curves $y = \ln x$, $y = -\ln x$, $x = e$. (2 points)

Total score: 20 points