Excercises
Matematics A1a
Differentiation

1. Find the derivatives of these functions:

a.) $y = 5^{2x+1}\left(x^3 + \tanh x\right)$, b.) $y = \dfrac{1 - \arcsin x}{1 + \arccos x}$, c.) $y = \dfrac{1}{2}\ln\dfrac{x+1}{x-1}$,

d.) $y = \cos\sqrt{2x - \sin 2x}$, e.) $y = \arctan^2 \dfrac{1}{x}$, f.) $y = \dfrac{\sqrt[3]{\tan 3x^2}}{(1-3x)^5}$.

g.) $y = (\ln x)^{\lg x}$, h.) $y = (1+x)^{1-x}$.

2. Find an equation for the tangent to the curve y = sin x at the point $\left(\dfrac{\pi}{4}, \sin\dfrac{\pi}{4}\right)$.

3. Find the equation of the lines that touch the curve $y = \sin^2 x$, $x \in \left[-\dfrac{\pi}{2}; \dfrac{\pi}{2}\right]$ and parallel to the line
$2y - \sqrt{3} \cdot x - 6 = 0$.

4. At which point(s) of the curve $y = xe^{-x^2}$ is the tangent of the curve horizontal?

5. At which points of the curve $y = \dfrac{x}{1-x^2}$ is the angle between the tangent of the curve and the x-axis equal to 45^{o}?

6.* Give the equations of the lines, that touch the curve $y = x^2 + 1$, and pass through the point (0,-1)!

7. Give all the extreme values of the function $y = \sin x + \cos x$ in the interval $[0;2\pi]$

8. Give the absolute maximum and the absolute minimum values of the function $y = \dfrac{10}{x^2+1}$ in the interval $[-1;2]$.

9. The function $f(x) = 9 - \dfrac{14}{x}$ is given. Find the value of *c* in the interval (2;7), so that

$$f'(c) = \frac{f(7) - f(2)}{7-2}.$$

10. Determine, whether Rolle's Theorem can be applied to function $f(x) = \dfrac{(x+2)(x-1)}{x-1}$ in the interval $[-2;0]$. If yes, give the value(s) of *c* in the given interval, so that $f'(c) = 0$. If the theorem can't be applied, explain, why not.

11. Find the open intervals, where the function *f*(*x*) is a.) rising; b.) falling; c.) concave down; d.) concave up:

$$f(x)=x^3-3x^2; \qquad f(x)=2x(x-4)^3; \qquad f(x)=\frac{x}{x^2+4}$$

12. Find the inflection points of the function $f(x)$:

a.) $f(x)=2x(x-4)^3$, b.) $f(x)=\dfrac{x}{x^2+4}$.

13. Give the limits of the following functions:

a.) $\lim\limits_{x\to 0}\sqrt{x}\ln x$, b.) $\lim\limits_{x\to\frac{\pi}{4}}\dfrac{\mathrm{tg}x-1}{\sin 4x}$, c.) $\lim\limits_{x\to\infty}\dfrac{\frac{\pi}{2}-\arctan x}{\sin\frac{1}{x}}$, d.) $\lim\limits_{x\to 0^+}(\mathrm{tg}\,x)^x$,

e.) $\lim\limits_{x\to 0}\left(\dfrac{1}{x}-\dfrac{1}{x^2}\right)$, f.) $\lim\limits_{x\to 0}\dfrac{5e^{x/5}-(x+5)}{x^2}$.

14. Graph the following functions:

a.) $y=\cos^2 2x$, b.) $y=\ln\sin\dfrac{x}{2}$, c.) $y=e^{-(x+1)^2}$, d.) $y=(x-2)^2(x+2)^2$,

e.) $y=\arcsin(\sin x)$, f.) $y=\dfrac{2x}{x-1}$, g.) $y=\dfrac{(x+2)^2}{(x-2)^2(x+2)}$, h.) $y=x^2e^{-x^2}$.