Exercises VI.

Mathematics A1a

2008 Nov – Dec.

INDEFINITE INTEGRALS:

1. $\int \frac{x^3-6x+5}{x}dx$,
2. $\int \frac{\sqrt[3]{x^2}-\sqrt[4]{x}}{x}dx$
3. $\int \frac{2\cdot 5^x-5\cdot 2^x}{5^x}dx$
4. $\int (2x+5)^{3/2}dx$
5. $\int \sinh(5x-7)dx$
6. $\int \frac{dx}{\cos^2(4-3x)}$
7. $\int \frac{dx}{4-3x}$
8. $\int \frac{e^{7x+1}}{e^{2x}}dx$
9. $\int \sqrt{5-2x}dx$
10. $\int \frac{x^2}{3+x^3}dx$
11. $\int \frac{dx}{x\ln x}$
12. $\int \cot 2xdx$
13. $\int \frac{2x+3}{x^2+3x}dx$
14. $\int \frac{e^{3x}}{5+12e^{3x}}dx$
15. $\int \frac{\sin x}{1-\cos x}dx$
16. $\int \sin^3 x\cdot\cos xdx$
17. $\int x\sqrt{3-x^2}dx$
18. $\int \frac{(\arctan x)^2}{1+x^2}dx$
19. $\int \frac{\sqrt[3]{\tan x}}{\cos^2 x}dx$
20. $\int \frac{3x-1}{x^2-2x+10}dx$
21. $\int \frac{3x-1}{x^2-2x-3}dx$
22. $\int \frac{3}{x^2+2x-15}dx$
23. $\int \frac{xdx}{\sqrt{4x^2+8}}$
24. $\int \frac{x-1}{\sqrt{3-2x-x^2}}dx$
25. $\int \frac{3x+2}{\sqrt{x^2-6x+25}}dx$
26. $\int \frac{10}{x^2-5x}dx$
27. $\int \frac{1+2x^2}{x^2\left(1+x^2\right)}dx$
28. $\int x\cdot e^{-x}dx$
29. $\int x\sin x\cos xdx$
30. $\int (x-2)\cos 3xdx$
31. $\int (x^2-1)\sinh(-x)dx$
32. $\int \ln 10xdx$
33. $\int x^2\ln xdx$
34. $\int \arccos\frac{x}{2}dx$
35. $\int (\sinh x+\cosh x)e^{-x}dx$
36. $\int x\sin^2 xdx$
37. $\int \frac{2^{3x}}{1+2^{3x}}dx$
38. $\int \sqrt{36-x^2}dx$ $(x=6\sin t)$
39. $\int \sqrt{25-4x^2}dx$
40. $\int \frac{4x}{\sqrt{4x+6}}dx$ $(\sqrt{4x+6}=t)$
41. $\int \sqrt{2x^2-8}dx$ $(x=2\cosh t)$
42. $\int \frac{x^2-4x+7}{x-2}dx$
43. $\int \frac{e^x}{e^{-x}+3}dx$ $(e^x=t)$
44. $\int \frac{4}{e^{2x}-4}dx$ $(e^x=t)$
45. $\int \arctan\sqrt{x}dx$ $(\sqrt{x}=t)$
46. $\int e^{\sqrt{x}}dx$ $(\sqrt{x}=t)$
47. $\int x\cdot e^{-x^2}dx$
48. $\int \cos^2 2xdx$
49. $\int \sin^2 x\cos^2 xdx$
50. $\int \sin^2 x\cos xdx$
51. $\int e^{\sin x}\cos xdx$
52. $\int \cos^4\frac{x}{2}dx$
53. $\int \sin 3x\sin 8xdx$
54. $\int \cos x\cos 5xdx$
55. $\int \cos^3 xdx$
56. $\int \cos^3 x\sin^2 xdx$

57. Give the following definite integrals:

a.) $\int_0^{\pi} \sin 2x \cos 2x dx$, b.) $\int_1^2 \frac{\sqrt{\ln x}}{x} dx$ c.) $\int_{-\pi/4}^{\pi/4} x \sin x dx$, d.) $\int_1^e \ln^2 x dx$,

e.) $\int_0^3 \frac{x}{\sqrt{25-x^2}} dx$, f.) $\int_{-2}^2 \frac{|x|}{1+x^2} dx = ?$

58. Find the area bounded by the following curves:

a.) x-axis, line $x = 2$, curve $y = \sqrt{x}$;

b.) $y = e^x$, $y = e^{-x}$, $x = 1$;

c.) $y = e^x$, $y = e^{-x}$, $y = e$

d.) $y = x^2, y = \frac{x^2}{2}, y = 2x$

e.) $y = \ln x$; its tangent, that goes through the origin; x-axis.

59. The region in the a.) part of the previous exercise is revolved about the x-axis. Give the surface area and the volume of the generated solid.

60. The parametric equations of an astroid are $x = a \sin^3 t$, $y = a \cos^3 t$, $0 \le t \le 2\pi$.
Give the area of the region bounded by the curve, the arc length of the curve, the surface area and the volume of the solid, generated by revolving the curve about the x-axis.

61. a.) Find the centroid of the region bounded by the curve $y = \sin \frac{x}{2}$, $0 \le x \le 2\pi$

b.) Find the volume of the solid generated by revolving the this curve about the x-axis.

62. Find the arc length of the curve $y = \frac{2}{3}(x-8)^{3/2}$ over the interval $[8;16]$.

63. Find the centroid of the region bounded by the following curves:

a.) $y = \sqrt[3]{x}, y = 0, x = 8$, b.) $y = x^2, y = 9$, $x = 0$, $x \ge 0$

64. Find the following improper integrals:

a.) $\int_2^{\infty} \frac{1}{x^3} dx$, b.) $\int_0^2 \frac{1}{x^3} dx$, c.) $\int_0^{\infty} 2^{-x} dx$, d.) $\int_0^{\infty} xe^{-2x} dx$,

e.) $\int_{-2}^2 \frac{dx}{\sqrt{4-x^2}} dx$, f.) $\int_{-\infty}^{-1} \frac{1}{x^2-5x} dx = ?$ g.) $\int_{-\infty}^{\infty} \frac{3}{4x^2+1} dx$