Excercises
Matematics A1a
Vectors in 3-space

1. Find the angle made by the two vectors: $\mathbf{u} = (2,1,0)$, $\mathbf{v} = (1,2,-1)$.

2. Write vector $\mathbf{u}$ as sum of components parelel and perpendicular to vector $\mathbf{v}$:
$$\mathbf{u} = (0;3;4), \quad \mathbf{v} = (1;1;0).$$

3. Given the points $P(1;-1;2)$, $Q(2;0;1)$ and $R(0;2;1)$.
a.) Find the area of the triangle PQR.
b.) Find a unit vector perpendicular to the plane PQR.

4. a.) Find the volume of the pyramid made by the points: $A(1;1;1)$, $B(2;3;0)$, $C(-2;1;4)$ and $D(1;8;-3)$.
b.) Find the distance from point D to the plane ABC.

5. Find the parametric equation system of the line that
a.) passes through the point $(1;2;5)$ and paralel to the z-axis.
b.) Passes through the point $(2;4;5)$ and perpendicular to the plane $3x + 7y - 5z = 21$.

6. Find the equation of the plane that
a.) passes through the point $A(1;-2;1)$ and perpendicular to the vector from O to A.
b.) passes through the points $(1;1;-1)$, $(2;0;2)$ and $(0;-2;1)$.

7. Find the distance of the point and the line:
a.) $P(2;1;3)$, $e: x = 2 + 2t, y = 1 + 6t, z = 3$;
b.) $P(3;-1;4)$, $e: x = 4 - t, y = 3 + 2t, z = -5 + 3t$.

8. a.)* Show that the following lines intersect.
b.) Find the equation of their plane.
$$e_1 : x = 1 + t, y = 2 + t, z = 1 - t; \qquad e_2 : x = 1 - 4t, y = 1 + 2t, z = 2 - 2t.$$

9. Find the distance of the point $P(2;-3;4)$ and the plane $x + 2y + 2z = 13$.

10. Find the angle of these planes in radians: $2x + 2y + 2z = 3$ és $2x - 2y - z = 5$.

11. Find the intersection point of the line $-x + 1 = \dfrac{y}{3} = z - 1$ and the plane $2x - y + 3z = 6$.

12. Find the points in which the line $\dfrac{x-1}{2} = -y - 1 = \dfrac{z}{3}$ meets the coordinate planes.

13. a.) Find the distance of the planes S1: $x + 2y + 6z = 1$ and S2: $x + 2y + 6z = 10$.
b.) Find the reflection of the plane S1 about the plane S2.

14. Find the intersection line of the planes S1: $x + y = 3$ and S2: $2y + z = 7$.

15.* Find the distance of the skew lines: $\dfrac{-x-5}{8} = \dfrac{y-7}{10} = \dfrac{z-10}{6}$ and $\dfrac{x-5}{3} = \dfrac{y+3}{5} = \dfrac{z-15}{4}$.

Answers:

1. 0,75 rad

2. $\left(\dfrac{3}{2};\dfrac{3}{2};0\right)+\left(-\dfrac{3}{2};\dfrac{3}{2};4\right)$

3. a.) $\sqrt{6}$ b.) $\pm\dfrac{1}{\sqrt{6}}(1;1;2)$

4. a.) 4   b.) $\dfrac{2\sqrt{2}}{3}$ 5. a.) $x=1, y=2, z=t$ b.) $x=3t+2, y=7t+4, z=-5t+5$

6. a.) $x-2y+z=6$ b.) $7x-5y-4z=6$ 7. a.) 0   b.) $\dfrac{9\sqrt{42}}{7}$

8. $y+z=3$ 9. 3   10. 1,76 rad   11. $\left(\dfrac{3}{2};-\dfrac{3}{2};\dfrac{1}{2}\right)$

12. $\left(0;-\dfrac{1}{2};-\dfrac{3}{2}\right)$, (-1;0;-3), (1;-1;0)   13. a.) $\dfrac{9}{\sqrt{41}}$ b.) $x+2y+6z=19$

14. $x=1+t, y=2-t, z=3+2t$ 15. $\sqrt{75}$