Double integrals
Mathematics A2
12th week

1. Sketch the region of integration and evaluate the integral:

a. $\int_0^{\pi}\int_0^{1} x\sin x\,dydx$ b. $\int_1^{\ln 8}\int_0^{\ln y} e^{x+y}\,dxdy$ c. $\int_0^{4}\int_{\sqrt{y}}^{4-\frac{y}{2}} dxdy$

d. $\iint_R (x+y+2)dA$, where R is the polygon with vertices (1,0), (1,1), (0,2), (-1,1), (-1,0).

2. Sketch the region of integration, interchange the order of integration and evaluate the integral:

$$\int_0^{8}\int_{\sqrt[3]{x}}^{2} \frac{1}{y^4+1}\,dydx$$

3. Find the improper integrals:

a. $\int_1^{\infty}\int_{e^{-x}}^{1} \frac{1}{x^3 y}\,dydx$ b. $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \frac{1}{(x^2+1)(y^2+1)}\,dxdy$

4. Using polar coordinates find the value of the double integrals:

a. $\iint_R \frac{1}{x^2+y^2+1}\,dA$ where $R=\left\{(x,y)\middle|1\le x^2+y^2\le 2, x\ge 0\right\}$,

b. $\iint_R e^{-(x^2+y^2)}dA$ where $R=\left\{(x,y)\middle|0\le x^2+y^2\le 1, x-y\ge 0\right\}$,

c. $\iint_R e^{-(x^2+y^2)}dA$ where $R=\left\{(x,y)\middle|-\infty<x<\infty, 0<y<\infty\right\}$

5. Find the volume of the solid bounded by the surfaces: $z=x^2+y^2, z=16$