

Fourier series
Mathematics A2
3rd week

Find the Fourier series associated with the following functions:

$$1. \quad f(x) = \begin{cases} 0, & 0 \leq x \leq \pi \\ 2, & \pi < x < 2\pi \end{cases} \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$2. \quad f(x) = \begin{cases} 0, & -\frac{\pi}{2} < x \leq \frac{\pi}{2} \\ 1, & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \end{cases} \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$3. \quad f(x) = \begin{cases} x, & 0 < x \leq \pi \\ 2\pi - x, & \pi < x \leq 2\pi \end{cases} \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$4. \quad f(x) = \begin{cases} \frac{\pi}{2} - x, & -\frac{\pi}{2} < x \leq \frac{\pi}{2} \\ -\frac{\pi}{2} + x, & \frac{\pi}{2} < x \leq \frac{3\pi}{2} \end{cases} \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$5. \quad f(x) = |2x|, \quad -\pi \leq x < \pi \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$6. \quad f(x) = x, \quad -\pi \leq x < \pi \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

$$7. \quad f(x) = \begin{cases} \sin x, & 0 \leq x \leq \pi \\ 0, & \pi < x < 2\pi \end{cases} \quad \text{otherwise } f(x) = f(x + 2k\pi).$$

8. Based on the former results find the sum of the series:

$$a.) \quad \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}$$

$$b.) \quad \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$$

Find the Fourier series associated with the following functions:

$$9. \quad f(x) = \cos^2 x$$

$$10. \quad f(x) = \sin^2 x \cdot \cos x$$

$$11. f(x) = \sin 5x \cdot (\cos x + \sin 3x)$$

$$12. f(x) = \sin^3 x$$

$$13. f(x) = \begin{cases} 0, & 0 \leq x < 1 \\ 1, & 1 \leq x < 2 \end{cases} \quad \text{otherwise } f(x) = f(x + 2k).$$

$$13. f(x) = \begin{cases} x, & 0 < x \leq 1 \\ 2 - x, & 1 < x \leq 2 \end{cases} \quad \text{otherwise } f(x) = f(x + 2\pi).$$