Systems of linear equations, matrix operations
Mathematics A2
4th week

1. For which value of *k* will the following system have no solution? Exactly one solution? Infinitely many solutions?

$$\begin{aligned} x - \ \ y &= 3 \\ 2x - 2y &= k \end{aligned}$$

2. Using Gaussian elimination solve the following systems:

a.)
$$\begin{aligned} x_1 + \ x_2 \ + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - \ 7x_2 + 4x_3 &= 10 \end{aligned}$$

b.)
$$\begin{aligned} 3x_1 + \ 2x_2 \ - x_3 &= -15 \\ 5x_1 + 2x_2 + \ 2x_3 &= 0 \\ 3x_1 + \ \ x_2 + \ 3x_3 &= 11 \\ 11x_1 + 7x_2 \qquad &= -29 \end{aligned}$$

c.)
$$\begin{aligned} 5x_1 + \ 2x_2 \ + 6x_3 &= 0 \\ -2x_1 \ + x_2 + 3x_3 &= 0 \end{aligned}$$

d.)
$$\begin{aligned} 2x_1 + \ x_2 \ + 3x_3 &= 0 \\ x_1 \ + \ 2x_2 \qquad &= 0 \\ x_2 + \ \ x_3 &= 0 \end{aligned}$$

e.)
$$\begin{aligned} 2x_1 - 4x_2 + x_3 + x_4 &= 0 \\ x_1 - 5x_2 + 2x_3 \qquad &= 0 \\ -2x_2 - 2x_3 - x_4 &= 0 \\ x_1 - 3x_2 \qquad - x_4 &= 0 \\ x_1 - 2x_2 - x_3 - x_4 &= 0 \end{aligned}$$

2. For which value of *a* will the following system have no solution? Exactly one solution? Infinitely many solutions?

$$\begin{aligned} x + 2y - 3z &= 4 \\ 3x - y + 5z &= 2 \\ 4x + y + (a^2 - 14)z &= a + 2 \end{aligned}$$

3. Find the value of *a* and *b* such that the system have
a.) one unique solution,
b.) infinitely many solutions,
c.) no solution:

$$\begin{aligned} ax_1 \qquad + bx_3 &= 2 \\ ax_1 + ax_2 + 4x_3 &= 4 \\ ax_2 + 2x_3 &= b \end{aligned}$$

2. Let $\mathbf{A}=\begin{bmatrix}1 & 0\\ 2 & 3\end{bmatrix}$, compute (if possible)

a.) $\mathbf{A}^3=$ b.) $\mathbf{A}^2-2\mathbf{A}+\mathbf{I}=$ c.) $\mathbf{A}\cdot\mathbf{A}^T-\mathbf{A}^T\cdot\mathbf{A}=$

d.) $\mathbf{A}^{-1}=$ e.) $\left(\mathbf{A}^T\right)^{-1}-\left(\mathbf{A}^{-1}\right)^T=$

3. Given the matrices find (if possible):

$$\mathbf{A}=\begin{bmatrix}3 & 0\\ -1 & 2\\ 1 & 1\end{bmatrix},\ \mathbf{B}=\begin{bmatrix}4 & -1\\ 0 & 2\end{bmatrix},\ \mathbf{C}=\begin{bmatrix}1 & 4 & 2\\ 3 & 1 & 5\end{bmatrix},\ \mathbf{D}=\begin{bmatrix}1 & 5 & 2\\ -1 & 0 & 1\\ 3 & 2 & 4\end{bmatrix},\ \text{and}\ \mathbf{E}=\begin{bmatrix}6 & 1 & 3\\ -1 & 1 & 2\\ 4 & 1 & 3\end{bmatrix}$$

a.) $\mathbf{D}+\mathbf{E}$
b.) $\mathbf{D}-\mathbf{E}$
c.) $3\mathbf{D}+5\mathbf{E}$
d.) $\mathbf{DE}$
e.) $\mathbf{ED}$
f.) $3\mathbf{C}-\mathbf{D}$
g.) $(\mathbf{AB})\mathbf{C}$
h.) $\mathbf{A}(\mathbf{BC})$
i.) $\mathbf{B}\mathbf{A}^T-\mathbf{C}^T$
j.) $\mathbf{D}^T\mathbf{E}^T-(\mathbf{ED})^T$

4. Find 2x2 matrices **A** and **B** such that

a.) $\left(\mathbf{A}+\mathbf{B}\right)^2=\mathbf{A}^2+2\mathbf{AB}+\mathbf{B}^2$ b.) $\left(\mathbf{A}+\mathbf{B}\right)^2\neq\mathbf{A}^2+2\mathbf{AB}+\mathbf{B}^2$