

**Functions of several variables,
partial derivatives, local extrema
Mathematics A2
10th week**

1. Given the function $f(x, y) = \ln(x^2 - y^2)$

- a.) find the domain and describe the level curves,
- b.) find the gradient vector of f at the point $(4;1)$,
- c.) find all second order partial derivatives of f .

2. Find the derivative of f at the point P in the direction of $\mathbf{u}$:

$$f(x, y) = x \arctan \frac{y}{x} \qquad P(1;1) \qquad \mathbf{u} = (2; -1).$$

3. Find the direction in which the function increases most rapidly at the given point. Find the derivative of f in this direction:

$$f(x, y, z) = xe^y + z^2 \qquad P(1; \ln 2; \frac{1}{2}).$$

4. Find the equation of the tangent plane of the function at P :

$$z = \sqrt{3 - x^2 - y^2} \qquad P(1;1;1).$$

5. Test the function for maxima, minima and saddle points. Find the maximum and minimum values of f :

$$f(x, y) = 9x^3 + \frac{y^3}{3} - 4xy$$