Function series, Power series, Taylor series
Mathematics A2
2nd week

1. For which real x values will the function series be convergent?

a.) $\sum_{n=0}^{\infty}(1+\ln x)^n$ b.) $\sum_{n=0}^{\infty}\frac{nx^n}{n+2}$ c.) $\sum_{n=0}^{\infty}\frac{x^n}{(2n)!}$

d.) $\sum_{n=1}^{\infty}\frac{(4x-5)^{2n+1}}{n^{3/2}}$ e.) $\sum_{n=0}^{\infty}\left(\frac{x}{x+2}\right)^n$

2. For which real x values will the following series be convergent?

$$1-\frac{1}{2}(x-3)+\frac{1}{4}(x-3)^2-\frac{1}{8}(x-3)^3+...+\left(-\frac{1}{2}\right)^n(x-3)^n+...$$

What is the sum of this series? Which series can be obtained by term-by-term differentiation?

3. Find the Taylor series generated by the given function centered at the given point:

a.) $y=e^{-x},\ a=0$ b.) $y=\ln(1+x),\ a=0$ c.) $y=\ln x,\ a=1$

d.) $y=\arctan\frac{x}{2},\ a=0$ e.) $y=x^2-2x+3,\ a=0$ f.) $y=x^2,\ a=1$

g.) $y=\frac{2}{x},\ a=2$

4. Give the McLaurin series of the functions:

a.) $y=xe^{2x}$ b.) $y=\frac{1}{(1-x)^2}$

5. Give the Taylor-series generated by the function $f(x)$ centered at $x=a$ and give the interval of convergence.

a.) $f(x)=\frac{1}{3+x},\ a=1$ b.) $f(x)=\sin^2 x,\ a=0$

6. Give the first four nonzero terms of the binomial series generated by the following functions:

a.) $y=(1-x)^{-1/2}$ b.) $y=(1+x^3)^{-1/2}$ c.) $y=(1-2x)^3$

7. Estimate the integral with an error of magnitude less than $\varepsilon=0,001$:

a.) $\int_0^{0,2}\frac{e^{-x}-1}{x}dx$ b.) $\int_0^{1}\sin(x^2)dx$ c.) $\int_0^{0,25}\sqrt[3]{1+x^2}\,dx$