Orthogonal diagonalization, quadratic forms
Mathematics A2
9th week

Take-home quiz
Due: April 13, 2012

1. Find a matrix **P** that orthogonally diagonalizes the matrix $\mathbf{A} = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$.

2. Do the matrix multiplication:

$$\mathbf{x}^T\mathbf{A}\mathbf{x} = \begin{bmatrix} x_1 & x_2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} =$$

3. Write the following quadratic forms in $\mathbf{x}^T\mathbf{A}\mathbf{x}$ form where **A** is a symmetric matrix:

a.) $x_1^2 + x_2^2 + 4x_1x_2$
b.) $9x_1^2 - x_2^2 + 4x_3^2 - 8x_1x_2 + 4x_1x_3$

4. Diagonalize the matrix of corresponding quadratic form and then in an (x_1, x_2) coordinate system sketch the graph of the equation:

a.) $x_1^2 + x_2^2 + 4x_1x_2 = 1$
b.) $5x_1^2 + 5x_2^2 + 8x_1x_2 = 9$