Sample Final Exam
Mathematics A2
May, 2011

1. a.) (3 points) Put down Leibniz criteria for the convergence of alternating numerical series.

b.) (4 points) Can we select the coefficients a_n of the power series $\sum_{n=1}^{\infty} a_n x^n$ such that the series

(1) converges for every $x \in \mathrm{R}$,
(2) converges absolutely for every $x \in \mathrm{R}$,
(3) converges conditionally for every $x \in \mathrm{R}$,
(4) has no point of convergence.

If your answer is YES, show an example, if your answer is NO, give reason in each case.

2. (8 points) Answer if the following statement is true or false. If you say TRUE, give a proof, if you say FALSE, give a counterexample.

(1) A set of two vectors in R^3 can be a linearly independent system.
(2) A set of two vectors in R^3 can be a basis for R^3.
(3) A set of four vectors in R^3 can be a linearly independent system.
(4) A set of four vectors in R^3 can be a basis for R^3.

3. a.) (3 points) Give the definition of the double integral of the function $f(x, y)$ over the region R.

b.) (3 points) Is it possible that the value for the integral $I = \iint_R e^{(x^2+y^2)^2} dxdy$ is $10 \le I \le 20$ over the region $\{R : (x; y), x^2 + y^2 \le 1\}$? Give reason for your answer. (You are not required to evaluate the integral.)

4. (7 points) Find the first 4 nonzero terms of the Fourier-series of the periodic function:

$$f(x) = \pi - |x|, \text{ if } -\pi < x \le \pi, \text{ otherwise } f(x + 2k\pi) = f(x).$$

5. (6 points) For which value(s) of *k* will the system be consistent? Solve the system with this *k*.

$$\begin{aligned} x - 2y + 3z - 4w &= 4 \\ y - z + w &= -3 \\ x + 3y - 3w &= k \\ -7y + 3z + w &= -3 \end{aligned}$$

6. (7 points) Give the maximum value of the function $f(x, y) = 3x + 4y$ subjected to the constrain $x^2 + y^2 = 25$.

7. (6 points) Find the value of the double integral $\iint_R (x+2y)dxdy$ over the triangular region with vertices A(0;0), B(1;1) and C(2;0).

8. (6 points) Find the value of the double integral $\iint_R \frac{\ln\left(1+\sqrt{x^2+y^2}\right)}{\sqrt{x^2+y^2}}dxdy$ over the region $\{R:(x;y),\ 1 \le x^2+y^2 \le 4, x-y \ge 0\}$.

9. (7 points) Find the triple integral of $f(x,y,z)=1$ over the 3-dimensional region defined by the conditions $0 \le z \le 4-\sqrt{x^2+y^2}, x^2+y^2 \le 1$.

Total score: 60 points
Passing limit: 24 points total