Vector Spaces
Mathematics A2
7th week

1. Which of the following sets are vector spaces. Give reason for your answer. If the set is a vector space, find its dimension and a basis for the space.
a.) three dimensional vectors in which at least one of the components is zero;
b.) three dimensional vectors in which the sum of the components is zero;
c.) 2x2 matrices having zeros along the main diagonal;
d.) 3x3 upper triangular matrices.

2. For which values of λ will the following three vectors make a linearly independent set?

$$\mathbf{a}_1\left(\lambda,-\frac{1}{2},-\frac{1}{2}\right) \qquad \mathbf{a}_2\left(-\frac{1}{2},\lambda,-\frac{1}{2}\right) \qquad \mathbf{a}_3\left(-\frac{1}{2},-\frac{1}{2},\lambda\right)$$

3. Show that for any vectors $\mathbf{u},\mathbf{v},\mathbf{w}\in\mathbb{R}^n$ the following set of three vectors make a linearly dependent system:

$$\mathbf{u}-\mathbf{v},\mathbf{v}-\mathbf{w},\mathbf{w}-\mathbf{u}$$

4. Find the value of *a* such that the $r(\mathbf{A})=2$ (if possible):

$$\mathbf{A}=\begin{bmatrix} 2 & 4 & 2 \\ -1 & -2 & -1 \\ 3 & 5 & 1 \\ -2 & 1 & 8 \\ 4 & 7 & a \end{bmatrix}$$

5. Find the dimension and a basis for the solution space of the following linear system:

$$\begin{bmatrix} -1 & 1 & -1 & -1 & -1 \\ -2 & 1 & -3 & -4 & -5 \\ -1 & 3 & 1 & 3 & 3 \end{bmatrix}\cdot\mathbf{x}=\mathbf{0}$$

6. Give the coordinates of the vector $\mathbf{v}(0,1,3)$ with respect to the basis formed by the vectors $\mathbf{a}_1(1,1,0)$, $\mathbf{a}_2(1,0,1)$, and $\mathbf{a}_3(0,1,1)$.