On the stability of relativistic computing devices (the strong cosmic censor conjecture)

Gábor Etesi

Budapest University of Technology and Economics
Department of Geometry
HUNGARY

Logic, Relativity and Beyond; Budapest, 25 August 2017

The meaning and physical formulation of the strong cosmic censor conjecture (**SCCC**)

We have the strong conviction that in the *classical* physical world at least, every physical event (possibly except the Big Bang) has a cause which is another and preceding physical event. R. Penrose' original aim in the 1960-1970's with formulating the strong cosmic censor conjecture (or hypothesis) was to protect this concept of causality in general gravitational situations. Mathematically speaking space-times having this property are *globally hyperbolic* therefore

SCCC. *A physically relevant (i.e., with a matter content subject to some energy condition), generic (i.e., stable) space-time is globally hyperbolic.*

Remark

By a *space-time* we will always mean an *inextendible*, connected, oriented and time-oriented 4-dimensional Lorentzian manifold.

(i) Without the "genericity" assumption the **SCCC** is trivially not true, almost all basic solutions (AdS, Taub–NUT, Gödel, Kerr–Newman, etc.) are counterexamples. But problem: what is "genericity"?

(ii) Conventional approach: using the initial value formulation "genericity" can be defined rigorously. Within this framework during the past five decades several partial results appeared and gave a support for the validity of the **SCCC** in certain classes of space-times (Gowdy space-times, space-times with compact Cauchy horizon, scalar field space-times, etc.).

The **SCCC** in four dimensions

(i) In the initial value approach "genericity" of three dimensional initial data (S, h, k) is defined rigorously and considered. However, when thinking about the **SCCC**, certainly one should be able to talk about the "genericity" of the four dimensional space-time (M, g) itself;

(ii) In light of discoveries in low dimensional differential topology during the 1980's typical smooth 4-manifolds have unexpected properties (called exotica) not detectable from a 3-manifold perspective. Therefore one has to be caucious when applies the genuinely 3 dimensional initial value paradigm, investigated during the 1950-60's, to genuinely 4 dimensional space-time problems;

(iii) The inherent three dimensionality of the initial value formulation fits well with the following important fact:

Theorem (Bernal–Sánchez 2003)

Let (M, g) be a globally hyperbolic space-time. Then there exists a diffeomorphism $M \cong S \times \mathbb{R}$ where S is a smooth 3-manifold. $\diamond$

expressing a similar inherent three dimensionality of the smooth structures underlying globally hyperbolic space-times. Hence it is not surprising that this solution method of the (vacuum) Einstein's equation produces affirmative answers for the **SCCC**;

(iv) But: every non-compact topological 4-manifold X admits uncountably many (with the cardinality of the continuum in ZFC set theory) smooth structures (Gompf, 1993) and most of these smooth structures are not of product form i.e., $X \ncong S \times \mathbb{R}$ with any 3-manifold S (exotica);

(v) Therefore, as it has been recognized recently, from the viewpoint of differential topology **SCCC** is very restrictive. For example:

Theorem (Chernov–Nemirovski 2013)

Let M be an m dimensional open contractible differentiable manifold and (M, g) be globally hyperbolic. Then $M \cong \mathbb{R}^m$. $\diamond$

but iff $m = 4$ there exist contractible 4-manifolds which are not diffeomorphic to the standard $\mathbb{R}^4$ (exotic or fake $\mathbb{R}^4$'s).

Summary: if the (vacuum) Einstein's equation does not admit solutions on such (not-smooth-product) 4-manifolds then exotica can be abandoned in physics but otherwise not. Besides the initial value formulation (always producing non-exotic solutions), however, there are yet other methods to solve the (vacuum) Einstein's equation, for example Penrose' non-linear graviton construction i.e., twistor theory.

Relativistic computation and the **SCCC**

How the **SCCC** is related to relativistic computation?

Definition

(M, g) is called a Malament–Hogarth space-time if there is a future-directed timelike half-curve $\gamma_C : [0, +\infty) \to M$ such that $\|\gamma_C\| = +\infty$ and a point $q \in M$ satisfying $\gamma_C([0, +\infty)) \subset J^-(q)$. The event $q \in M$ is called a Malament–Hogarth event.

Another timelike curve $\gamma_O : [0, b] \to M$ also exists such that $\|\gamma_O\| < +\infty$, $\gamma_O([0, b]) \subset J^-(q)$, $\gamma_O(0) = \gamma_C(0)$ and $\gamma_O(b) = q$. Then $(M, g, q, \gamma_C, \gamma_O)$ can be used to perform non-Turing computations (e.g. Hogarth, 1992, 1994; Etesi–Németi, 2002; Welch, 2008; etc.).

Theorem

Let (M, g) be a Malament–Hogarth space-time. Then (M, g) is not globally hyperbolic. $\diamondsuit$

In fact, being Malament–Hogarth and being non-globally hyperbolic is almost the same:

Theorem

Let (M, g) be a non-globally hyperbolic space-time. If it is moreover distinguishable then (M, g) is conformally equivalent to a Malament–Hogarth space-time. $\diamondsuit$

Remark

From now on we assume that a Malament–Hogarth space-time is physically reasonable as well i.e., (M, g) is a solution of the Einstein's equation with vacuum or a matter content satisfying some energy condition.

Consequently Malament–Hogarth space-times are subjects to the **SCCC** (= a physically relevant and generic space-time is globally hyperbolic): if it holds true then no physically relevant Malament–Hogarth space-time can be generic i.e., stable!

In other words: in order to make the idea of relativistic computation via Malament–Hogarth space-times physically realistic, first one has to invalidate the **SCCC**!

The problem of stability or genericity

One can put the question of stability into a more general context.

Definition

A quintuple $(M, g, q, \gamma_C, \gamma_O)$ is called a gravitational computer if (M, g) is a space-time, γ_C, γ_O are timelike curves and $q \in M$ is an event such that the images of the curves lie within $J^-(q)$.

Remark

This concept is broad enough to serve as an abstract model for any kind of artificial computing systems based on classical physics so that an artificial computing system can perform non-Turing computations iff the corresponding gravitational computer is defined in an ambient space-time (M, g) possessing the Malament–Hogarth property.

Case-by-case studies demonstrate that all known physically relevant solutions (M, g) of the Einstein constraint equations which are non-globally hyperbolic—i.e., arise as further extensions of maximal Cauchy developments of initial data sets (S, h, k)—are unstable: an arbitrary "small" but "generic" perturbation of their metric in the maximal Cauchy development destroys extendibility across the Cauchy horizon. That is, a generic perturbation of these non-globally hyperbolic space-times makes them globally hyperbolic.

This is in particular true for certain physically relevant Malament–Hogarth space-times: small perturbation of them (e.g. taking into account the backreaction of the computer itself on the geometry, etc.) destroys the Malament–Hogarth property.

That is, if **SCCC** is true then all physically relevant artificial computing systems based on Malament–Hogarth space-times i.e., capable for non-Turing computations contain an inherent instability.

Other artificial computing systems in principle capable for non-Turing computations (e.g. the generalized quantum computers of Calude–Pavlov, 2002; Kieu, 2003, 2004; etc.) also seem to suffer from inherent instabilities.

Even worse: artificial computing systems used just for very complex but still Turing computations (e.g. usual quantum computers or the Chern–Simons TQFT computer of Freedman, 1998 to calculate the Jones polynomial of knots) apparently contain an inherent instability increasing with the degree of computational complexity.

However none of these considerations take into account the aforementioned exotic phenomena; but this can significantly alter the situation at least in the case of global relativistic computations.

Recall that **SCCC** requires a space-time to have a product smooth structure while there are many smooth 4-manifolds which are not smooth products; moreover beyond the initial value formulation there is a twistorial method to solve Einstein's equation.

Definition of a counterexample to the SCCC

Definition

Let (M, g) be a maximal extension of the maximal Cauchy development $(D(S), g|_{D(S)})$ of an initial data set (S, h, k). The (continuous) Lorentzian manifold (M', g') is a perturbation of (M, g) relative to (S, h, k) if

(i) M' has the structure

$$M' := \text{the connected component of } M \setminus H \text{ containing } S$$

where, for a connected open subset $S \subset U \subsetneqq M$ containing the initial surface, the subset H is closed and satisfies $\emptyset \subsetneqq H \subsetneqq \partial U = \overline{U} \setminus U$ i.e., is a closed subset in the boundary of U;

continued...

...continued

(ii) g' is a solution of Einstein's equation at least in a neighbourhood of the initial surface $S \subset M'$ with a fundamental matter represented by a stress-energy tensor T' obeying the dominant energy condition at least in a neighbourhood of $S \subset M'$;

(iii) (M', g') does not admit further extensions and $(S, h') \subset (M', g')$ with $h' := g'|_S$ is a spacelike complete sub-3-manifold.

Definition

Let (M, g) be a maximal extension of the maximal Cauchy development $(D(S), g|_{D(S)})$ of an initial data set (S, h, k). Then (M, g) is a robust counterexample to the **SCCC** if it is very stably non-globally hyperbolic i.e., all of its perturbations (M', g') relative to (S, h, k) are not globally hyperbolic.

Remark

(i) Perturbations of the whole 4 dimensional space-time are considered;

(ii) The concept of a robust counterexample is logically stronger than the "generic" one.

Counterexamples to the **SCCC** via exotica

Strongly motivated by the Bernal–Sánchez smooth splitting theorem (2003) and the Chernov–Nemirovski smooth censorship theorem (2013) we consider a family of fake $\mathbb{R}^4$'s:

Theorem (Gompf 1993, Taubes 1987)

There exists a pair (R^4, K) consisting of a differentiable 4-manifold R^4 homeomorphic but not diffeomorphic to the standard $\mathbb{R}^4$ and a compact oriented smooth 4-manifold $K \subset R^4$ such that

(i) *R^4 cannot be smoothly embedded into the standard $\mathbb{R}^4$ i.e., $R^4 \nsubseteq \mathbb{R}^4$ but it can be smoothly embedded as a proper open subset into the complex projective plane i.e., $R^4 \subsetneqq \mathbb{C}P^2$;*

continued...

...continued

(ii) *Take a homeomorphism* $f : \mathbb{R}^4 \to R^4$, *let* $0 \in B^4_t \subset \mathbb{R}^4$ *be the standard open 4-ball of radius* $t \in \mathbb{R}^+$ *centered at the origin and put* $R^4_t := f(B^4_t)$ *and* $R^4_{+\infty} := R^4$. *Then*

$$\left\{ R^4_t \;\middle|\; r \leqq t \leqq +\infty \text{ such that } 0 < r < +\infty \text{ satisfies } K \subset R^4_r \right\}$$

is an uncountable family of nondiffeomorphic exotic $\mathbb{R}^4$*'s none of them admitting a smooth embedding into* $\mathbb{R}^4$ *i.e.,* $R^4_t \nsubseteqq \mathbb{R}^4$ *for all* $r \leqq t \leqq +\infty$. $\diamond$

Using this exotic family we arrive at (Etesi 2017)

$\overline{\textbf{SCCC}}$. *From every connected and simply connected closed (i.e., compact without boundary) smooth 4-manifold M one can construct an open (i.e., non-compact without boundary) smooth 4-manifold*

$$X_M = M \# \underbrace{\mathbb{C}P^2 \# \dots \# \mathbb{C}P^2}_{\textit{finitely many}} \#_K R^4 \tag{1}$$

and a smooth Ricci-flat Lorentzian metric g on it such that (X_M, g) is not globally hyperbolic. Moreover, any "sufficiently large" (in an appropriate topological sense) perturbation (X'_M, g') of this space cannot be globally hyperbolic, too.
This very stable non-global-hyperbolicity follows because X_M as a smooth 4-manifold contains a "creased end" (see Figure 1), a typical four dimensional phenomenon.

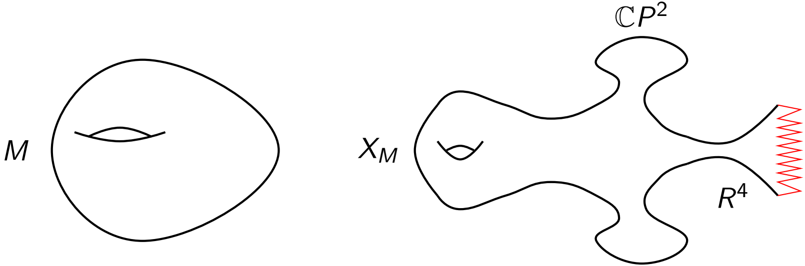

Figure 1. Construction of X_M out of M.
The creased end of X_M is drawn by a red zig-zag.

Idea of the construction.

(i) Following Taubes 1992, glue sufficiently many $\mathbb{C}P^2$'s to M to make it a self-dual 4-manifold and remove an S^2 from one of these $\mathbb{C}P^2$'s (note: $\mathbb{C}P^2 \setminus S^2 \cong R^4$) to get a space X_M;

(ii) Apply Penrose' non-linear graviton construction (i.e., twistor theory) to obtain a hyper-Kähler Riemannian metric g_1 on this X_M;

(iii) Use "Wick rotation" (in a precise way) to get a Ricci-flat Lorentzian metric g on X_M. $\diamond$

Concerning the very stable non-global-hyperbolicity:

Lemma (Etesi 2017)

Consider the Ricci-flat Lorentzian 4-manifold (X_M, g) as above with any spacelike and complete sub-3-manifold $(S, h) \subset (X_M, g)$ in it (non-empty submanifolds of this sort exist). Let (S, h, k) be the (necessary partial) initial data set inside (X_M, g) induced by (S, h) and let (X'_M, g') be a perturbation of (X_M, g) relative to (S, h, k). Consider the pair (R^4, K). Assume that X'_M contains the image, present in the R^4-factor of X_M in its decmoposition (1), of the compact subset K. Then (X'_M, g') is not globally hyperbolic.

Idea of the proof. Assume (X'_M, g') was globally hyperbolic. Then $X'_M \cong S \times \mathbb{R}$ which is impossible by its creased end. $\diamond$

Although (X_M, g) is not a robust counterexample as we defined, it is still very reasonable to consider the Ricci-flat Lorentzian spaces (X_M, g) as generic counterexamples to the **SCCC**.

Remark

The resolution of the apparent conflict between the initial value approach (providing affirmative answers for the **SCCC**) and this more global one (providing results against the **SCCC**) is that the initial value approach probes only the tubular neighbourhood of 3 dimensional spacelike submanifolds in the ambient 4 dimensional space-time hence cannot detect exotica at all!

What is the physical interpretation of the very stably non-globally-hyperbolic vacuum space-times (X_M, g) ? (They exist in a superabundance.)

How they can be used to construct stable non-Turing gravitational computers?

For further details please check:

G. Etesi: *A proof of the Geroch–Horowitz–Penrose formulation of the strong cosmic censor conjecture motivated by computability theory*, Int. Journ. Theor. Phys. **52**, 946-960 (2013), arXiv: `1205.4550v3 [gr-qc]`;

`http://www.math.bme.hu/~etesi/censor2.pdf`

G. Etesi: *Exotica and the status of the strong cosmic censor conjecture in four dimensions*, preprint, 24 pp. (2017), arXiv: `1707.09180 [gr-qc]`;

`http://www.math.bme.hu/~etesi/censor4.pdf`

Thank you!