

Gröbner theory of zero dimensional ideals with a view toward combinatorics

Abstract of the PhD thesis of Bálint Felszeghy

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Gröbner bases are special systems of generators of a polynomial ideal. A relatively new application of Gröbner theory is (algebraic) combinatorics, for which zero dimensional ideals are of great importance. We investigate in this topic: we contribute with theoretical examination of Gröbner bases of zero dimensional ideals, while giving equal emphasis to their combinatorial applications.

In the thesis, we introduce the Lex Game, a tool which allows one to easily compute the lexicographic standard monomials of certain zero dimensional ideals. The Lex Game shall help us to explore zero dimensional Gröbner theory by providing a way to obtain Gröbner bases of some families of ideals. We also formulate a theorem on the shape of the reduced Gröbner basis of zero dimensional ideals.

We apply the tools of Gröbner theory to provide easier proofs of known theorems, such as Harima's duality theorem, Alon's combinatorial Nullstellensatz, a generalization of the fundamental theorem of symmetric polynomials by Garsia and Wilson's rank formula for certain inclusion matrices. We also present new applications to extremal combinatorics.

The thesis is based on five papers of us, but some parts—at least in this generality—have not been published yet.