Addenda and Errata

On 29. November 2000., after discussions with József Balogh

1. A gap in the Aizenman–Lebowitz proof

In *Theorem 2.3 (b)* inequality (4) asserts some kind of stability of the function $\sigma(n,p)$, the existence of a "wide plateau". The proof of this in [AiL] is correct, but for $\hat{\sigma}(p)$ this argument implies only a lower bound by $\sup\{g(z)|z \geq A\}$, I think. This bound tends to 0 as $p \to \infty$, so we cannot say that $\hat{\sigma}(p)$ is between two absolute constants. Thus we have no good upper bound for $Q(p,n)$ in the critical regime, and so there is no good lower bound for the threshold $P(n)$.

Nevertheless, it seems to us that rearranging the ingredients of [AiL] may easily lead to a complete proof of the main theorem.

2. The sharp threshold

At the moment I have no idea why I thought inequality (7) in *Theorem 2.3 (e)* to be valid. In fact, the "sharp threshold" of $\lambda_c(p)$ proved in [AiL] gives no clue for the sharp threshold of $P(n)$.

It seems that the sharp threshold can really be proved by the *Bourgain-Friedgut theorem* [Bg], using the facts we know about "critical droplets"; this has been recently done by *J. Balogh* and *B. Bollobás*, though some details are not clear for me yet.

3. The threshold in higher dimensions

Our *Conjecture 2.1* has turned out to be far from the truth. In the 3-dimensional 3-neighbour bootstrap percolation the threshold is

$$P_{3,3}(n) = \Theta\left(\frac{1}{\log\log n}\right),$$

and a simple heuristic argument gives k-times iterated logarithm for $P_{k,k}(n)$; see **R. Cerf – E. Cirillo:** Finite size scaling in three-dimensional bootstrap percolation, *Ann. Probab.*, 27 (1999), no. 4, 1837–1850.