

$$\sin^2 x + \cos^2 x = 1$$

$$\sin(x \pm y) = \sin x \cos y \pm \sin y \cos x$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin y \sin x$$

$$\sin 2x = 2 \sin x \cos x, \quad \cos 2x = \cos^2 x - \sin^2 x$$

$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \cdot \operatorname{tg} y}, \quad \operatorname{tg} 2x = \frac{2 \operatorname{tg} x}{1 - \operatorname{tg}^2 x}$$

$$\sin x + \sin y = 2 \sin \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\sin x - \sin y = 2 \sin \frac{x-y}{2} \cos \frac{x+y}{2}$$

$$\cos x + \cos y = 2 \cos \frac{x+y}{2} \cos \frac{x-y}{2}$$

$$\cos x - \cos y = -2 \sin \frac{x+y}{2} \sin \frac{x-y}{2}$$

$$\sin x \sin y = -\frac{1}{2} [\cos(x+y) - \cos(x-y)]$$

$$\sin x \cos y = \frac{1}{2} [\sin(x+y) + \sin(x-y)]$$

$$\cos x \cos y = \frac{1}{2} [\cos(x+y) + \cos(x-y)]$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}, \quad \cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\operatorname{sh} x = \frac{e^x - e^{-x}}{2}, \quad \operatorname{ch} x = \frac{e^x + e^{-x}}{2}$$

$$\operatorname{ch}^2 x - \operatorname{sh}^2 x = 1$$

$$\operatorname{sh} 2x = 2 \operatorname{sh} x \operatorname{ch} x, \quad \operatorname{ch} 2x = \operatorname{ch}^2 x + \operatorname{sh}^2 x$$

$$\operatorname{sh}^2 x = \frac{\operatorname{ch} 2x - 1}{2}, \quad \operatorname{ch}^2 x = \frac{\operatorname{ch} 2x + 1}{2}$$

DIFFERENCIÁLÁSI SZABÁLYOK:

$$(cf)' = cf' \quad (c \text{ konstans})$$

$$(f+g)' = f' + g', \quad (fg)' = f'g + fg'$$

$$\left(\frac{f}{g}\right)' = \frac{f'g - fg'}{g^2}, \quad \frac{dy}{dx} = \frac{dy}{dt} \frac{dt}{dx}$$

$$(x^n)' = nx^{n-1} \quad (n \neq 0 \text{ valós konstans})$$

$$(e^x)' = e^x, \quad (a^x)' = a^x \ln a$$

$$(\sin x)' = \cos x, \quad (\cos x)' = -\sin x$$

$$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}, \quad (\operatorname{ctg} x)' = -\frac{1}{\sin^2 x}$$

$$(\sinh x)' = \cosh x, \quad (\cosh x)' = \sinh x$$

$$(\ln x)' = \frac{1}{x}, \quad (\log_a x)' = \frac{1}{x \ln a}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}, \quad (\arccos x)' = -\frac{1}{\sqrt{1-x^2}}$$

$$(\operatorname{arctg} x)' = \frac{1}{1+x^2}, \quad (\operatorname{arcctg} x)' = -\frac{1}{1+x^2}$$

$$(\operatorname{arsh} x)' = \frac{1}{\sqrt{1+x^2}}, \quad (\operatorname{arch} x)' = \frac{1}{\sqrt{x^2-1}}$$

$$(\operatorname{arth} x)' = \frac{1}{1-x^2}, \quad (\operatorname{arcth} x)' = -\frac{1}{x^2-1}$$

INTEGRÁLÁSI SZABÁLYOK:

$$\int af(x) dx = a \int f(x) dx \quad (a \text{ konstans})$$

$$\int [f(x) + g(x)] dx = \int f(x) dx + \int g(x) dx$$

$\int f(ax+b) dx = \frac{1}{a} F(ax+b) + c$, ahol F az f primitív függvénye

$$\int f^m(x) f'(x) dx = \frac{f^{m+1}(x)}{m+1} + c, \quad \text{ha } m \neq -1$$

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + c$$

$$\int f(x)g'(x) dx = f(x)g(x) - \int f'(x)g(x) dx$$

$$\int x^n dx = \frac{x^{n+1}}{n+1} + c \quad (n \neq -1), \quad \int \frac{1}{x} dx = \ln |x| + c$$

$$\int a^x dx = \frac{a^x}{\ln a} + c, \quad \int e^{ax} dx = \frac{1}{a} e^{ax} + c$$

$$\int \sin x dx = -\cos x + c, \quad \int \cos x dx = \sin x + c$$

$$\int \operatorname{tg} x dx = -\ln |\cos x| + c, \quad \int \operatorname{ctg} x dx = \ln |\sin x| + c$$

$$\int \frac{dx}{\cos^2 x} = \operatorname{tg} x + c, \quad \int \frac{dx}{\sin^2 x} = -\operatorname{ctg} x + c$$

$$\int \ln x dx = x \ln x - x + c$$

$$\int \frac{dx}{a^2 - x^2} = \begin{cases} \frac{1}{a} \operatorname{arth} \frac{x}{a} + c, & \text{ha } \left|\frac{x}{a}\right| < 1 \\ \frac{1}{a} \operatorname{arcth} \frac{x}{a} + c, & \text{ha } \left|\frac{x}{a}\right| > 1 \end{cases}$$

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \operatorname{arctg} \frac{x}{a} + c, \quad \int \frac{dx}{\sqrt{a^2 + x^2}} = \operatorname{arsh} \frac{x}{a} + c$$

$$\int \frac{dx}{\sqrt{a^2 - x^2}} = \operatorname{arcsin} \frac{x}{a} + c, \quad \int \frac{dx}{\sqrt{x^2 - a^2}} = \operatorname{arch} \frac{x}{a} + c$$

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}, \quad \binom{n}{k} = \binom{n}{n-k}, \quad \binom{n}{0} = 1, \quad (a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \dots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n = \sum_{k=0}^n \binom{n}{k}a^{n-k}b^k$$

$$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots + \frac{x^n}{n!} + \dots \quad (\text{minden } x\text{-re});$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^{n-1} \frac{x^{2n-1}}{(2n-1)!} + \dots \quad (\text{minden } x\text{-re});$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots + (-1)^n \frac{x^{2n}}{(2n)!} + \dots \quad (\text{minden } x\text{-re});$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots + x^n + \dots \quad (-1 < x < 1);$$

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + \dots + (-1)^n x^n + \dots \quad (-1 < x < 1);$$

$$\ln(x+1) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots + (-1)^{n+1} \frac{x^n}{n} + \dots \quad (-1 < x \leq 1);$$

Tetszőleges α valós szám esetén

$$(1+x)^\alpha = 1 + \frac{\alpha}{1!}x + \frac{\alpha(\alpha-1)}{2!}x^2 + \dots + \frac{\alpha(\alpha-1)\dots(\alpha-n+1)}{n!}x^n + \dots$$

($|x| < 1$, de α -tól függően más x értékekre is lehet konvergencia; ha α természetes szám, akkor a binomiális tételt kapjuk);

$$\ln \frac{1+x}{1-x} = 2 \left(x + \frac{x^3}{3} + \frac{x^5}{5} + \dots + \frac{x^{2n-1}}{2n-1} + \dots \right) \quad (|x| < 1);$$

$$\sqrt{1+x} = 1 + \frac{1}{2}x - \frac{1}{2 \cdot 4}x^2 + \frac{1 \cdot 3}{2 \cdot 4 \cdot 6}x^3 - \dots + (-1)^{n-1} \frac{1 \cdot 3 \cdot 5 \dots (2n-3)}{2 \cdot 4 \cdot 6 \dots 2n}x^n \dots$$

($|x| \leq 1$);

$$\frac{1}{\sqrt{1+x}} = 1 - \frac{1}{2}x + \frac{1 \cdot 3}{2 \cdot 4}x^2 - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6}x^3 + \dots + (-1)^n \frac{1 \cdot 3 \cdot 5 \dots (2n-1)}{2 \cdot 4 \cdot 6 \dots 2n}x^n \dots$$

($-1 < x \leq 1$);

$$\operatorname{ch} x = 1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \dots + \frac{x^{2n}}{(2n)!} + \dots \quad (\text{minden } x\text{-re});$$

$$\operatorname{sh} x = \frac{x}{1!} + \frac{x^3}{3!} + \frac{x^5}{5!} + \dots + \frac{x^{2n+1}}{(2n+1)!} + \dots \quad (\text{minden } x\text{-re});$$

Speciálisan a 2π szerint periodikus valós f függvény (2π szerinti) *Fourier-sorának* nevezzük a

$$\sum_{n=0}^{\infty} (a_n \cos nx + b_n \sin nx)$$

trigonometrikus sort, ha

$$a_0 = \frac{1}{2\pi} \int_0^{2\pi} f(x) dx \quad b_0 = 0$$

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos nx dx \quad b_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \sin nx dx \quad (n = 1, 2, 3, \dots)$$

EGYENLŐTLENSÉGEK:

$$(1+x)^n \geq 1+nx, \quad x \in (-1, \infty)$$

$$\frac{n}{\frac{1}{a_1} + \dots + \frac{1}{a_n}} \leq \sqrt[n]{a_1 \dots a_n} \leq \frac{a_1 + \dots + a_n}{n}, \quad a_1, \dots, a_n > 0$$

(egyenlőség pontosan $a_1 = \dots = a_n$ esetén)

$$x + \frac{1}{x} \geq 2 \quad \text{minden } x > 0 \text{ esetén}$$