Hausaufgaben 5.

Taylor-Reihen

Geben Sie die Potenzreihendarstellung um $x_0 = 0$ der folgenden Funktionen an:

1. $f(x) = \dfrac{7}{6-5x}$ $\quad (\dfrac{7}{6}\sum_{n=0}^{\infty}\left(\dfrac{5}{6}x\right)^n, \quad |x| < \dfrac{6}{5})$
2. $f(x) = \dfrac{5}{9-x^2}$ $\quad (\sum_{k=0}^{\infty}\dfrac{5}{9^{k+1}}x^{2k}, \quad |x| < 3)$
3. $f(x) = \sin 2x$ $\quad (\sum_{k=0}^{\infty}(-1)^k\dfrac{(2x)^{2k+1}}{(2k+1)!}, \quad -\infty < x < \infty)$
4. $f(x) = \sqrt{e^x}$ $\quad (\sum_{k=0}^{\infty}\dfrac{x^k}{2^k\,k!}, \quad -\infty < x < \infty)$
5. $f(x) = \ln(1+x^2)$ $\quad (\sum_{k=1}^{\infty}(-1)^{k+1}\dfrac{x^{2k}}{k}, \quad |x| < 1)$
6. $f(x) = \sqrt[3]{8+x}$ $\quad (2\sum_{k=0}^{\infty}\binom{\frac{1}{3}}{k}\left(\dfrac{x}{8}\right)^k, \quad |x| < 8)$
7. $f(x) = \dfrac{1}{\sqrt{1-x^2}}$ $\quad \sum_{n=0}^{\infty}\binom{-\frac{1}{2}}{n}(-x^2)^n, \quad -1 < x < 1)$

Fourier-Reihen

1. $f(x) = 1 - x,\ x \in (-\pi, \pi],\ f(x+2k\pi) = f(x)$

$(f(x) \approx 1 + \sum_{k=1}^{\infty}\frac{2}{k}(-1)^k \sin kx)$

2. $f(x) = \cos x,\ -\frac{\pi}{2} < x \le \frac{\pi}{2},\ f(x+k\pi) = f(x)$

$(f(x) = \frac{2}{\pi} + \frac{4}{\pi}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\cos 2nx}{4n^2-1})$

3. $f(x) = \operatorname{sgn}(\cos x),\ -\pi < x \le \pi,\ f(x+2k\pi) = f(x),$ (sgn = signum = Vorzeichen)

$(f(x) \approx \frac{4}{\pi}\sum_{n=0}^{\infty}(-1)^n\frac{\cos(2n+1)x}{2n+1})$

4. $f(x) = x,\ x \in (-1, 1],\ f(x+2k) = f(x)$

$(f(x) \approx \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n}\sin n\pi x)$

5.

$$f(x) = \begin{cases} x+1, & -1 < x \le 0 \\ -x+1, & 0 < x \le 1 \end{cases}, \quad f(x+2k) = f(x)$$

$(f(x) = \frac{1}{2} + \frac{4}{\pi^2}\sum_{n=1}^{\infty}\frac{\cos(2n-1)\pi x}{(2n-1)^2})$