Clinogonal axonometry

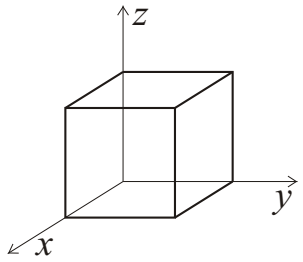
**Clinogonal axonometry**: here again, we project the figure in a given direction into a given plane, together with the coordinate system. If the direction of the projection is not perpendicular to the plane of projection, we write about *clinogonal axonometry*.

**Theorem of Pohlke.** *Any choice of the axonometric coordinate system and distortion ratios $q_x : q_y : q_z$ can be realized.* More specifically, for any axonometric coordinate system and distortion ratios there is a projection plane and a direction such that the projection of the original axonometric system is the given one, with the given distortion ratios. (Named after the German mathematician Karl Wilhelm Pohlke, 1810-1876.)

Consequently, we have much freedom in the choice of the axonometric coordinate system, and the distortion ratios. Nevertheless we should aim at choosing a pictorial projection.

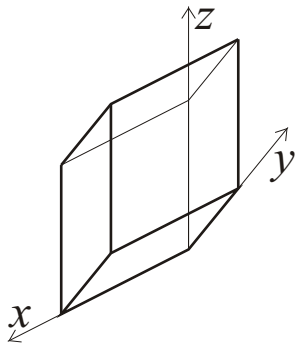
The projection of the unit cube:

$q_x : q_y : q_z = 31 : 50 : 50$



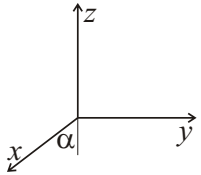

*pictorial*

$q_x : q_y : q_z = 19 : 11 : 20$



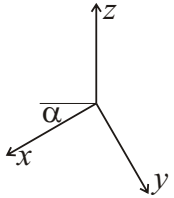

*not pictorial*

Frontal axonometry: $y \perp z$



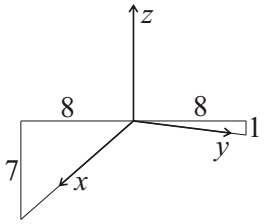

$\alpha = 30°, 45°, 60°$;
$q_x = 1/2, 2/3, 1$;
$q_y = q_z = 1$;
($\alpha = 45°$, $q_x = 1$: cavalier projection)

Horizontal axonometry: $x \perp y$



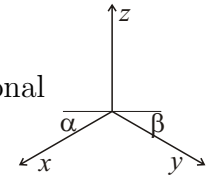

$\alpha = 30°, 45°, 60°$;
$q_x = q_y = 1$;
$q_z = 1/2, 2/3, 1$;

Conventional axonometry

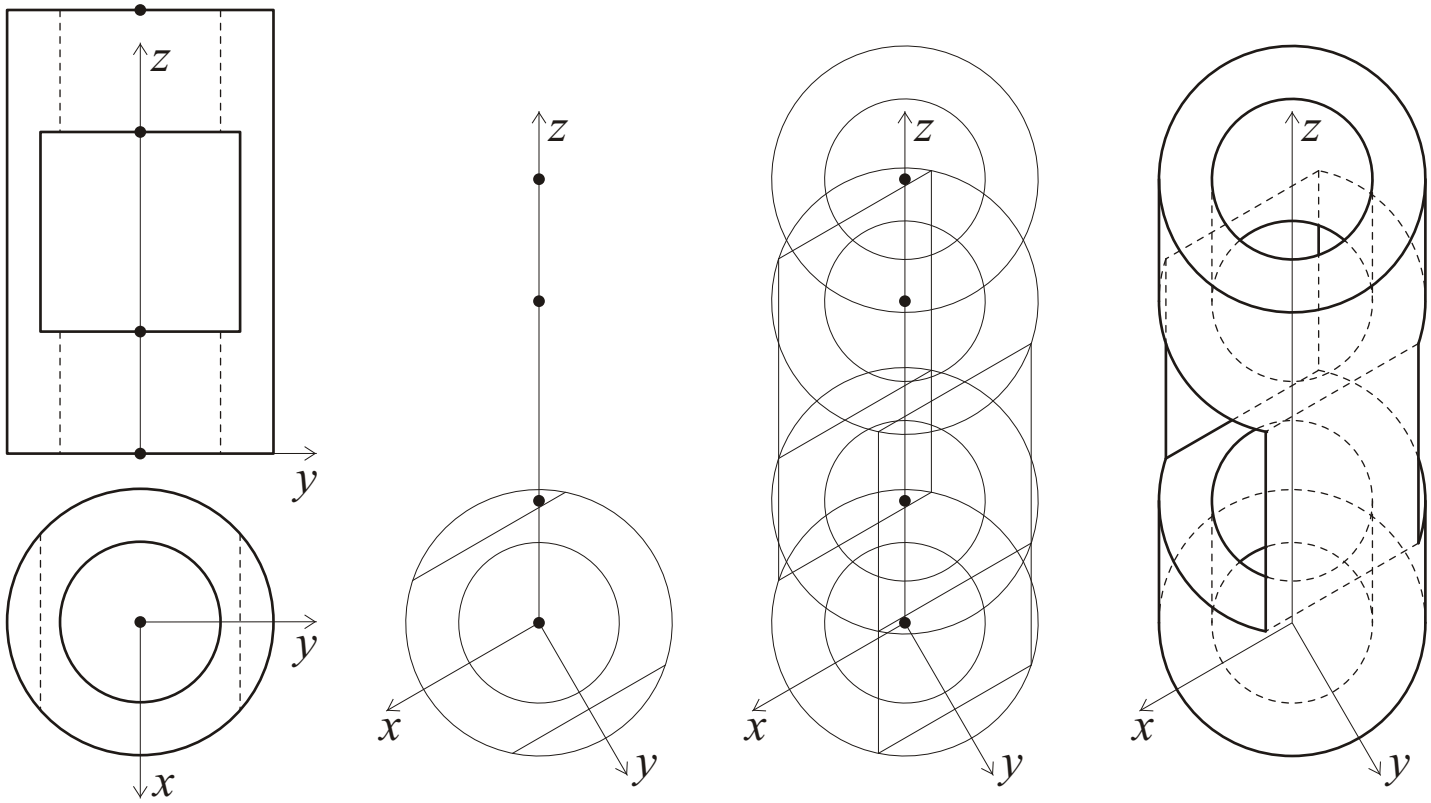



$q_x = 1/2$, $q_y = q_z = 1$
(approximately a resized orthogonal axonometric view)

Isometric axonometry




$\alpha = \beta = 30°$;
$q_x = q_y = q_z = 1$;
(a resized axonometric view)




**Exercise.** *Construct a horizontal axonometric view of the object given with its front and top views:* $\alpha = 30°$, $q_x = q_y = q_z = 1$.

Since $q_x = q_y$, and the projections of the $x$ and $y$ axes are perpendicular, the axonometric projection plane as parallel to the $[x, y]$ plane. Thus, the projection of the base is congruent to the top view (the projection of a circle on the top view is a circle). Thus, the top view of the base can be copied into the axonometric coordinate-system. Since $q_z = 1$, the different levels can be measured directly parallel to the $z$ axis, using the front view of the object.
Then we draw the edges of the object, and examine visibility.