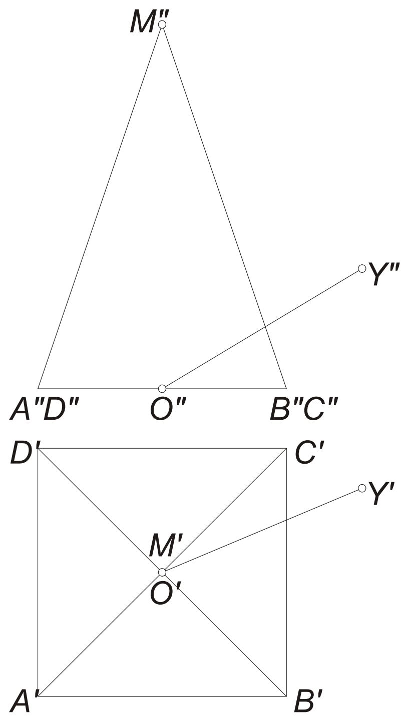
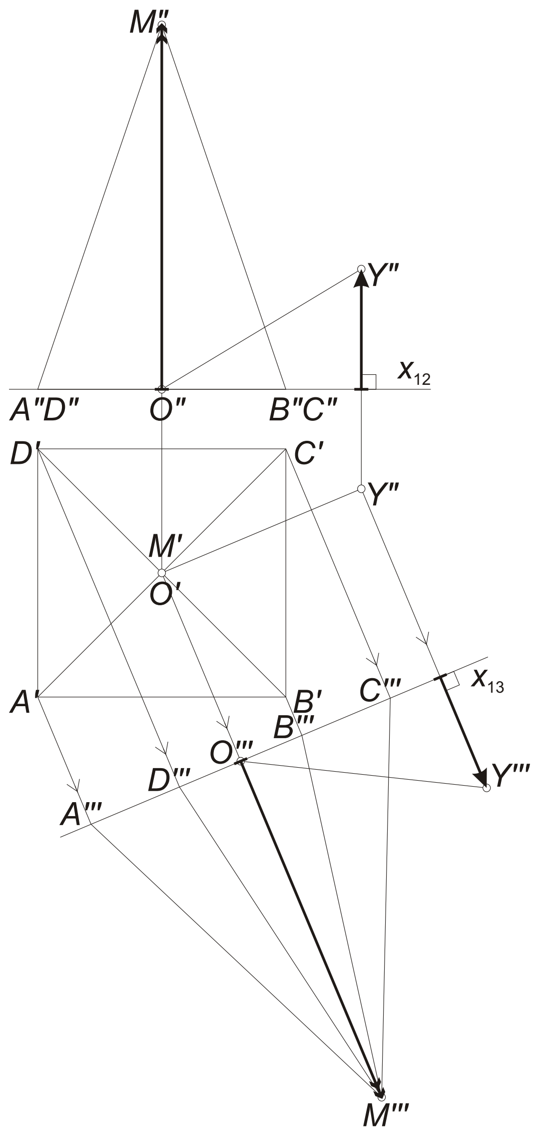
Transformation of a projection plane

View of a polyhedron in a given direction



Exercise. *A square-based $ABCDM$ pyramid is given with its front and top views, and also a line OY, where O is the centre of the base of the pyramid. Construct the projection of the pyramid onto a plane perpendicular to OY. Show the visibility of the solid, assuming that it is made of sheets, and its base is removed.*



To obtain the view of the pyramid in the given direction, using projection plane transformation, we need to introduce a projection plane perpendicular to OY. The projecting lines in this view are parallel to OY, and thus, to get the required view, our task is to transform OY into a projecting line.

For the transformation we fix the intersection x_{12} of the horizontal and the vertical projection planes, for example as the level of O''. Then the pyramid is standing on the horizontal projection plane.

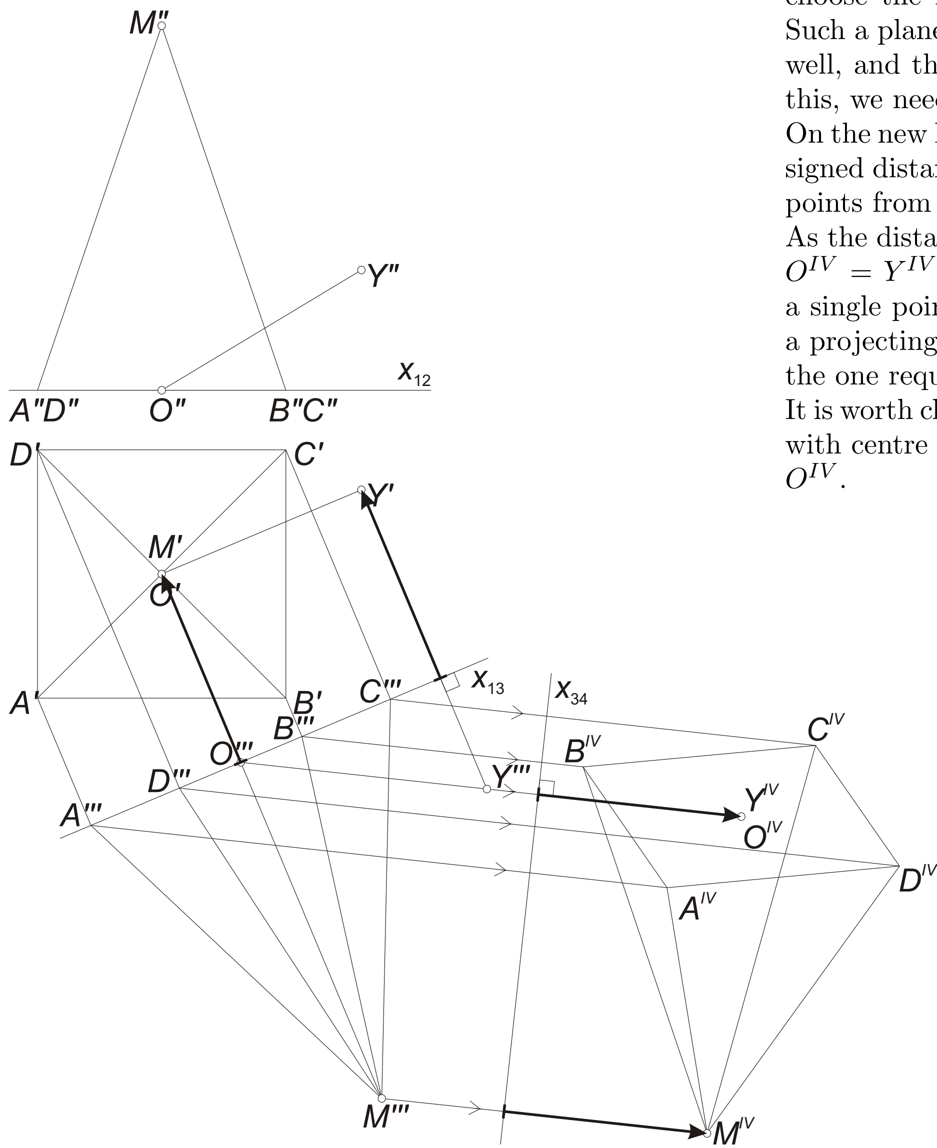
Since OY is parallel to neither the horizontal, nor the vertical projection plane, and thus, the new projection plane is not perpendicular to any of the original ones, we cannot carry out the transformation in one step.

Thus, first we transform OY into a parallel line. For this, we replace the vertical plane of projection with the new one and choose the third plane of projection parallel to OY. That is, let $x_{13}||O'Y'$, and otherwise arbitrary.

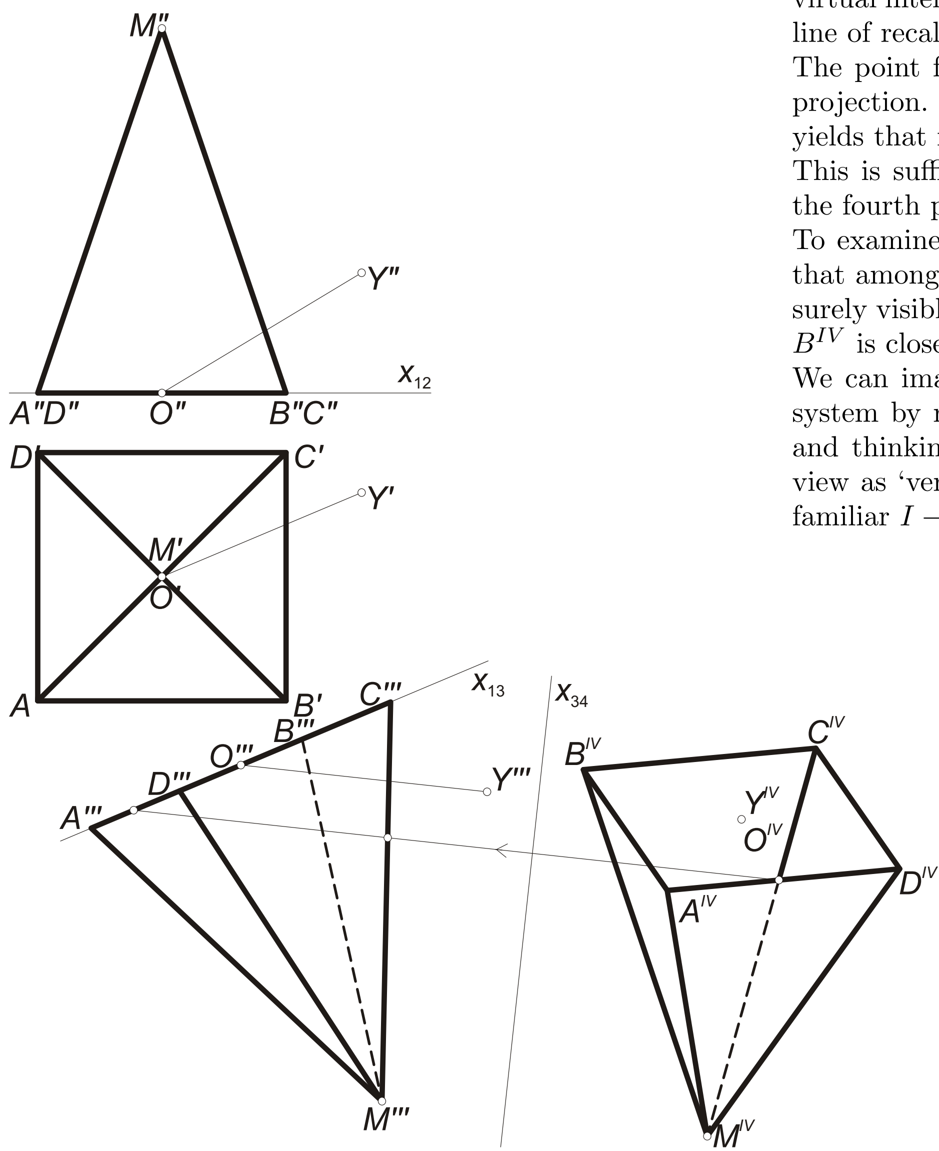
We transform O, Y and the vertices of the pyramid. On the new lines of recall, we measure, from the new axis x_{13}, the signed distances of the vertical projections of the transformed points from the old axis x_{12}. (Note that the vertical projections are omitted in the new $I-III$ system.) Since the points A, B, C, D, O are on the horizontal plane of projection, their vertical projections are on x_{12}, and thus their third projections will be on x_{13}.

The result of the transformation is that in the new system, OY is is a parallel line.

We note that, alternatively, the transformation can be carried out in such a way that the horizontal view is replaced with the new one, in which case $x_{23}||O''Y''$.



Since OY is parallel to the third plane of projection, we may choose the fourth plane of projection perpendicularly to OY. Such a plane is perpendicular to the third plane of projection as well, and thus, they form a valid projection plane system. For this, we need to choose the axis x_{34} perpendicularly to $O'''Y'''$. On the new lines of recall, we measure, from the new axis x_{34}, the signed distances of the horizontal projections of the transformed points from the old axis x_{13}.
As the distances of O' and Y' from x_{13} are equal, it follows that $O^{IV} = Y^{IV}$, and thus, the fourth projection of the line OY is a single point. This means that in the $III - IV$ system OY is a projecting line, and that the view we have just constructed is the one required in the exercise.
It is worth checking if the fourth projection of the square $ABCD$, with centre O, is a parallelogram $A^{IV}B^{IV}C^{IV}D^{IV}$, with centre O^{IV}.



To examine the visibility in the fourth view, we examine the virtual intersection point of the edges AD and MC. Drawing its line of recall, we obtain the third projections of the two points. The point farther from x_{34} covers the other one in the fourth projection. In our case it holds for the point on AD, which yields that near this virtual intersection point, AD covers MC. This is sufficient to determine the visibility of the pyramid in the fourth projection.

To examine the visibility in the third view, we should observe that among the points, D^{IV} is farthest from x_{34} and that M is surely visible. Thus the edge MD is visible. On the other hand, B^{IV} is closest to x_{34}, which yields that MB is not visible.

We can imagine the examination of visibility in the $III - iV$ system by rotating the drawing paper to make x_{34} horizontal, and thinking of the third view as ‘horizontal’, and the fourth view as ‘vertical’. Then we can determine visibility line in the familiar $I - II$ system.