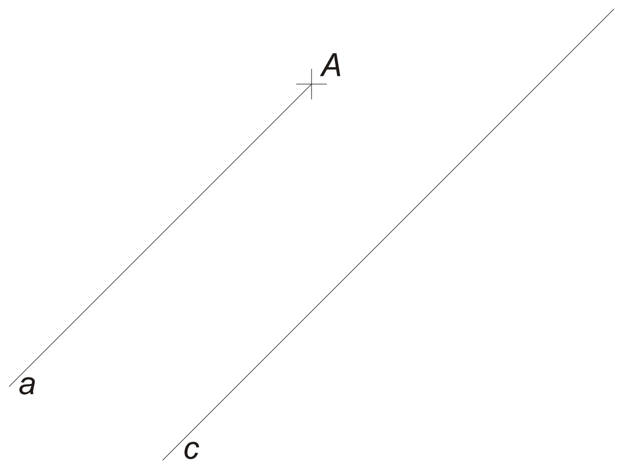
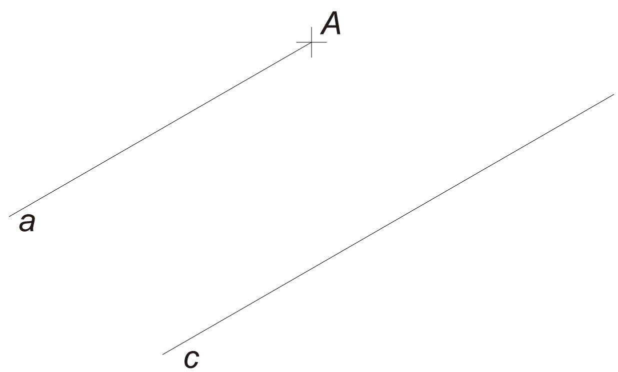
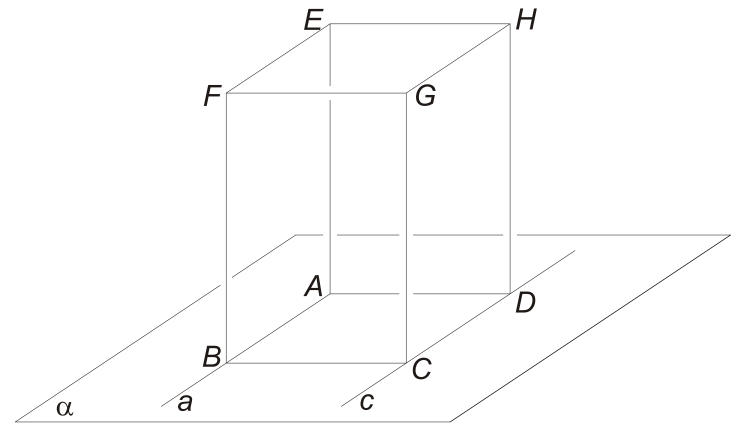
Metric constructions

Construction of a square-based, regular prism with its base lying in an oblique plane



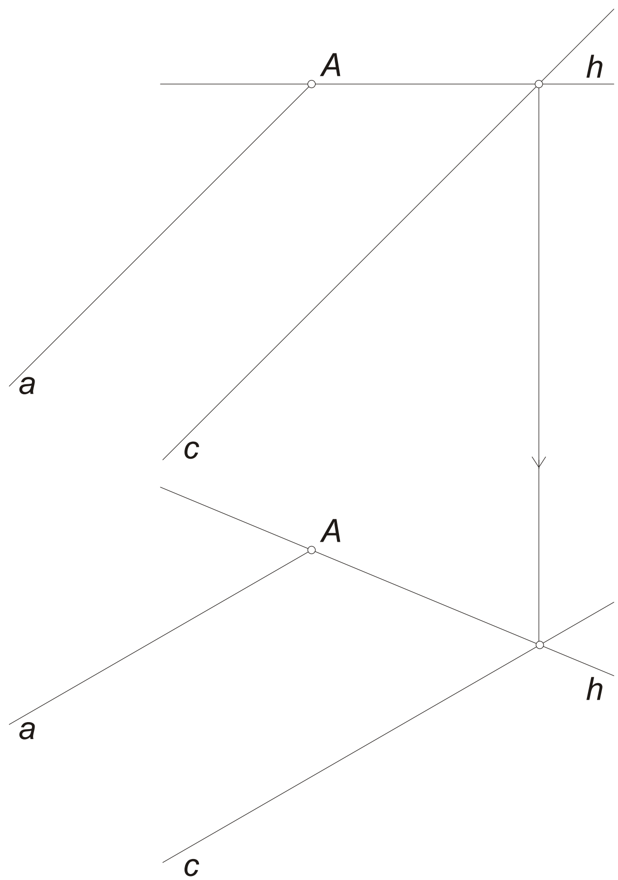


Exercise. *Given a half line a, starting at A, and a line c parallel to a. Construct a square-based, regular prism ABCDEFGH with base ABCD, if the edge AB lies on a, CD lies on c, and its height is one and the half times the side-length of the base. Show visibility, assuming that the prism is a solid body.*

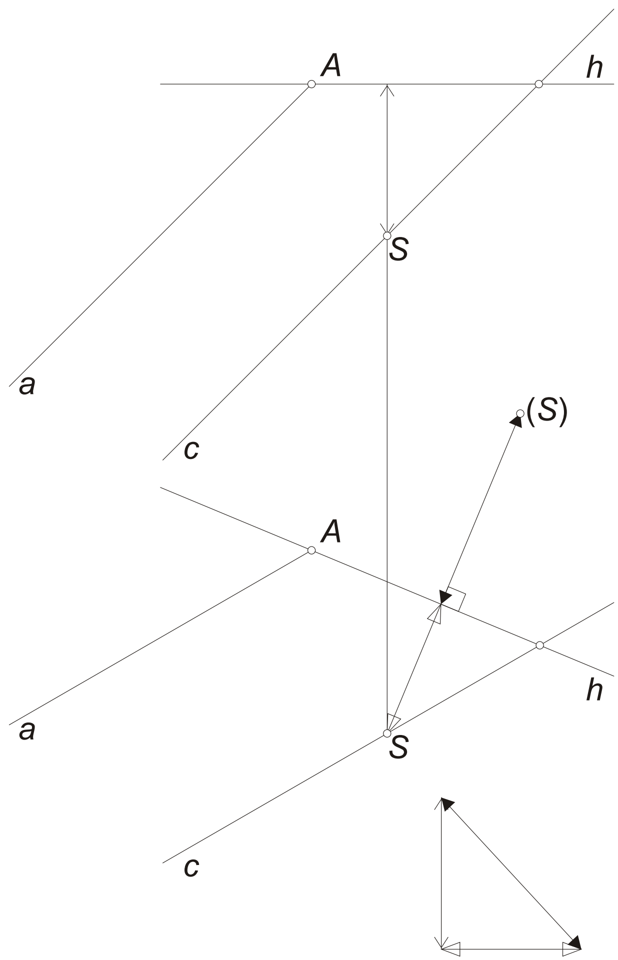


We make a plan for the construction using the fundamental steps of metric constructions.

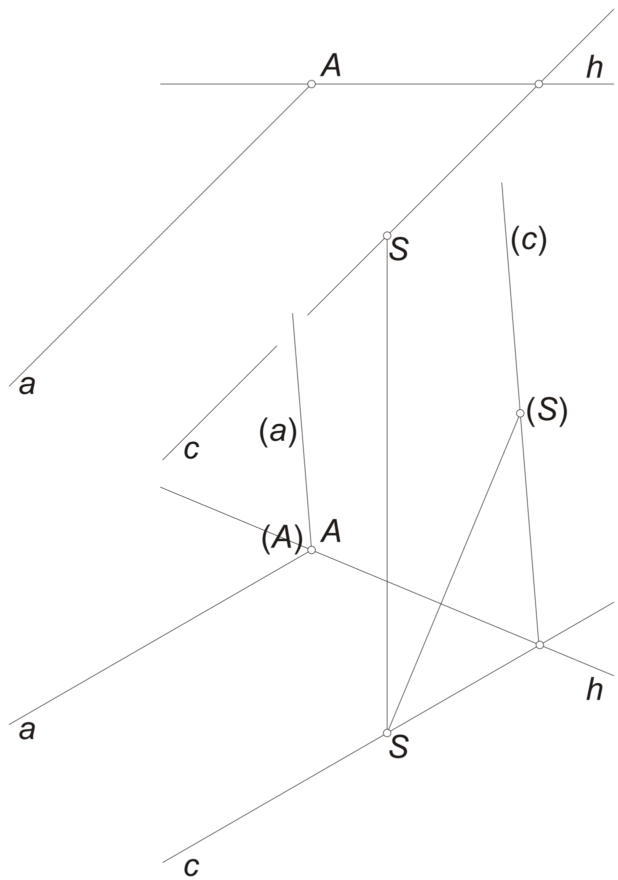
(1) We rotate the plane α of the base into a horizontal position. In the rotated plane we construct the base, then rotate back to obtain the two projections of the base.
(2) We construct the lines, passing through the vertices of the base, perpendicular to α.
(3) We measure the height of the prism on these lines to obtain the vertices of the roof of the prism. For the real size of the side-length of the square we use its rotated copy, from which we construct the real size of the height.



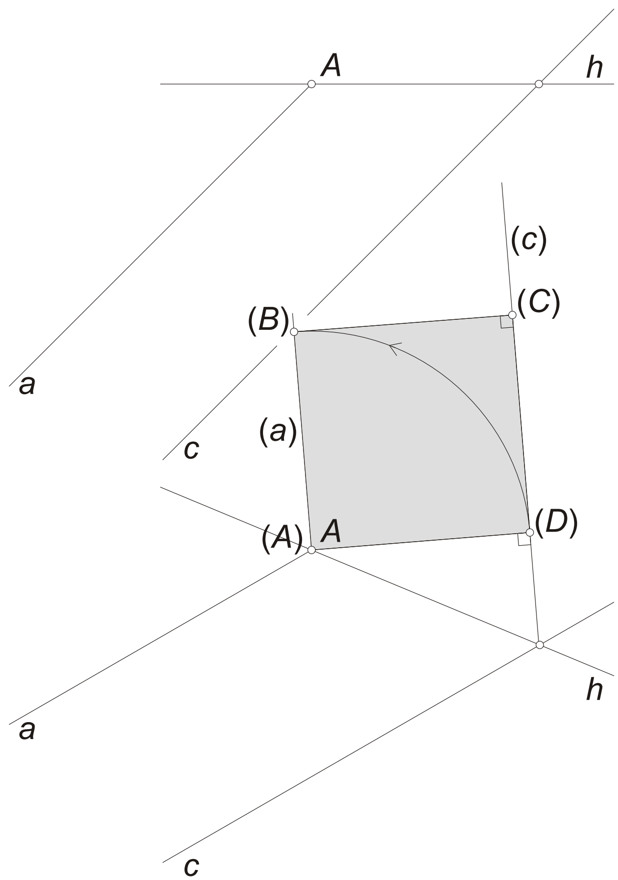
To rotate the plane of the base, we choose a horizontal line of the plane as the axis of the rotation. We can choose, for example, the line h passing through A.



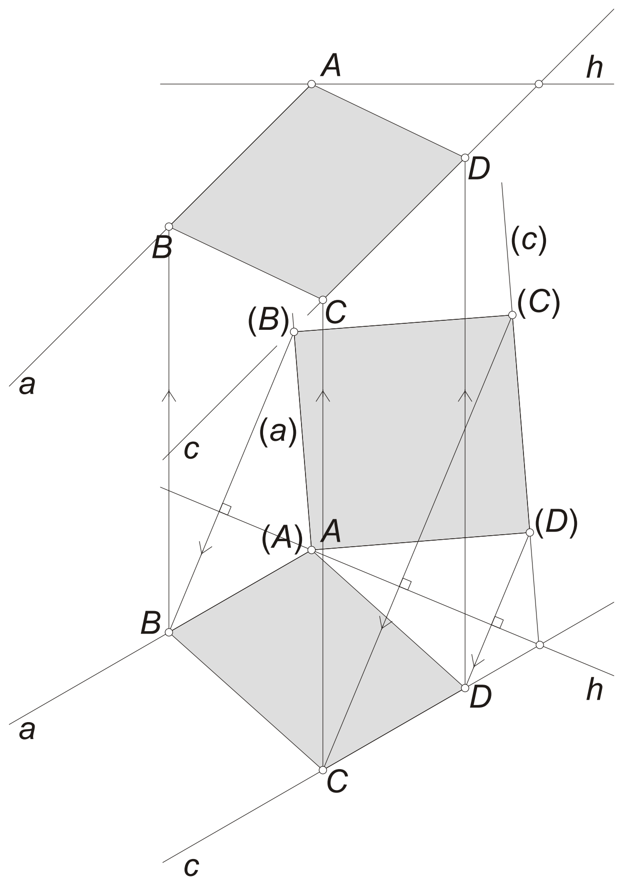
We choose a point of the plane that does not lie on h, for example, the point S of c.
During the rotation, S moves on a circle, contained in a horizontal projecting plane, and thus, its horizontal projection moves on the line, perpendicular to h', and passing through S'.
The radius of this circle is the distance of S and h, which we can construct using a difference triangle: its horizontal leg is the distance of S' and h', its vertical leg is the difference of the levels of S'' and h'', and then its hypotenuse is the distance of S and h. This is measured, from h', on the line perpendicular to h', to obtain the rotated copy (S) of S.



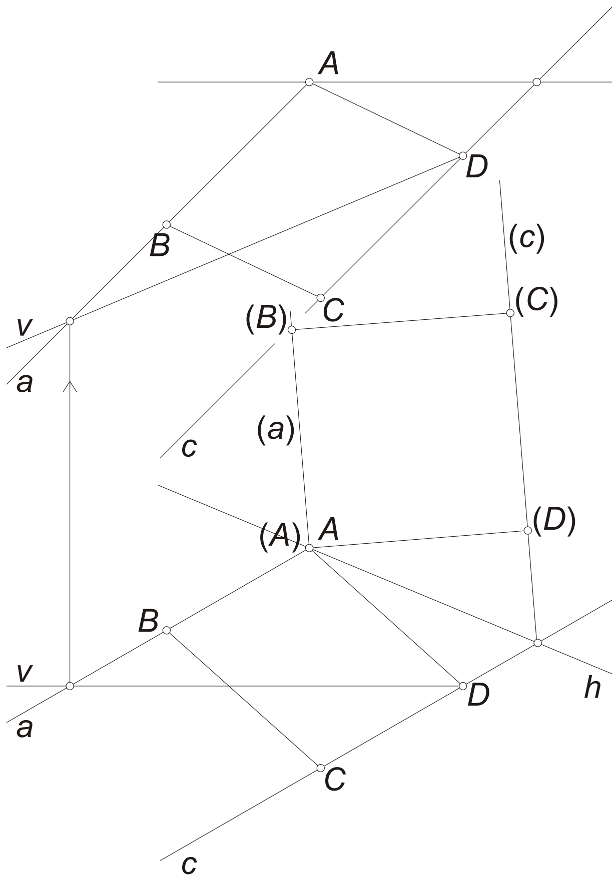
During the rotation, the points of the axis are fixed points, and thus A, and the intersection point of h and a. By connecting the latter one we obtain the rotated copy (c) of c. To construct the rotated copy (a) of the half line a, we draw a half line parallel to (c), starting at $A' = (A)$.



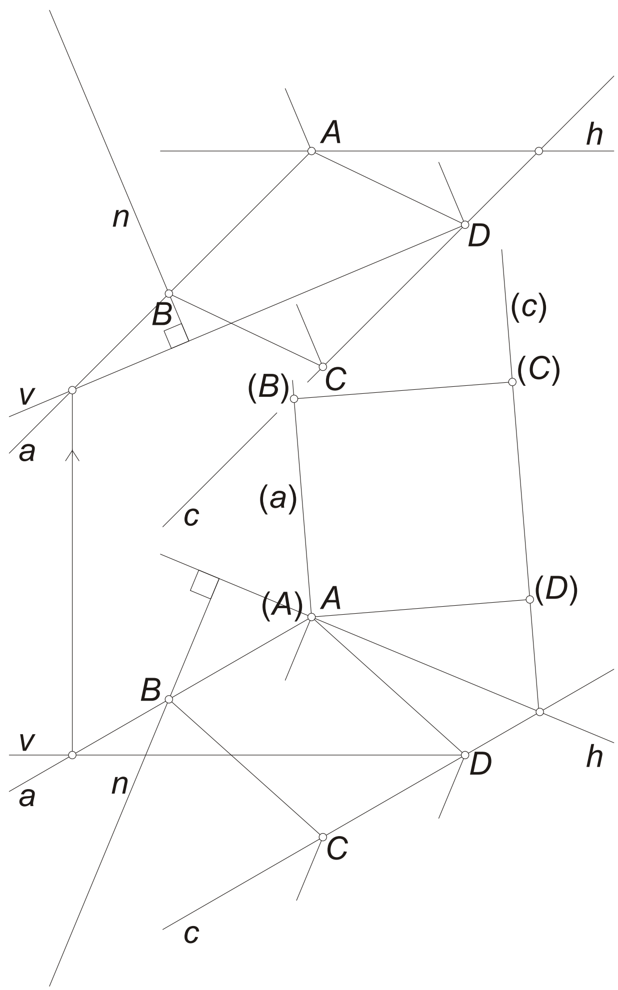
In the rotated plane we construct the square $(A)(B)(C)(D)$.



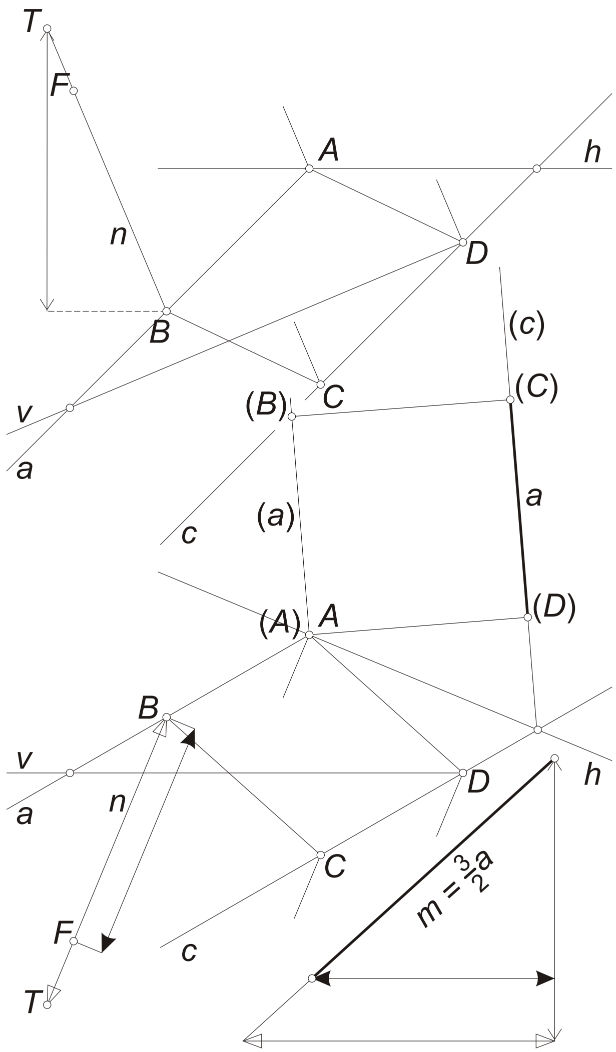
We rotate back the plane and construct the horizontal and the vertical projections of the square base.



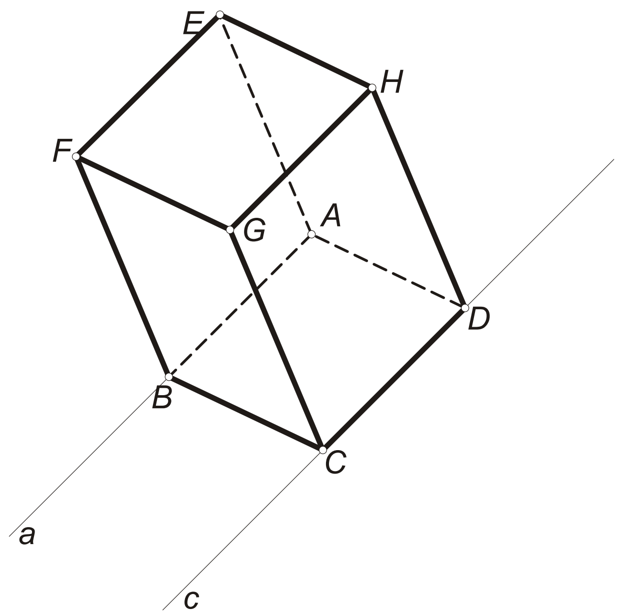
To construct a line perpendicular to a plane we need to know a horizontal and a vertical line of the plane. We already know a horizontal line, h, of the plane of the square, now we choose a vertical line in it. For example, we take the vertical line v, passing through D.

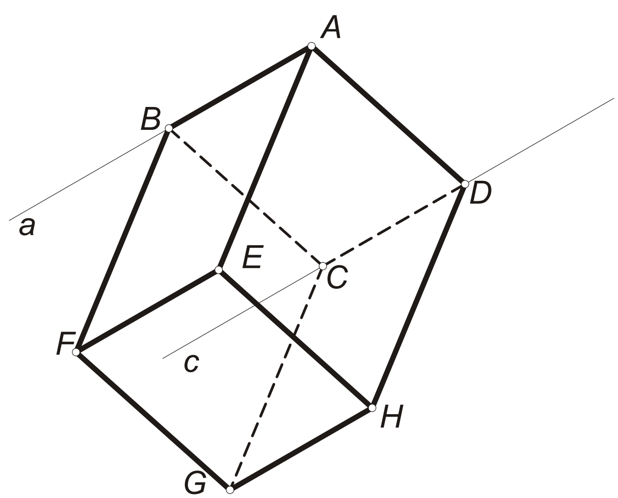


The horizontal/vertical projections of the sidelines containing the lateral edges of the prism are perpendicular to the horizontal/vertical projections of the horizontal/vertical lines of the plane of the base, respectively. For example, n is the line containing BF, and the other lines are parallel to n.



To measure the length of the lateral edges we take an arbitrary auxiliary point T on n, different from B, and construct the difference triangle of the segment BT. The length of the horizontal leg is the distance of B' and T', whereas the length of the vertical one is the difference of the levels of B'' and T''.
We rescale the difference triangle in such a way that its hypotenuse, the length of which shows the real distance of the points, is equal to the required height: one and a half times the side-length of the base. The side-length is shown in the rotated plane, say, $(A)(D)$ is a side of the square.
The length of the horizontal leg of the rescaled triangle shows the length of the horizontal projections of the lateral edges. This should be measured on n' from B' to obtain the horizontal projection F' of the other endpoint of the corresponding edge.





Using the fact that the lateral edges are parallel, and thus their projections are of equal length, we can readily draw the remaining edges of the prism.
Finally, we show the visibility of the solid.