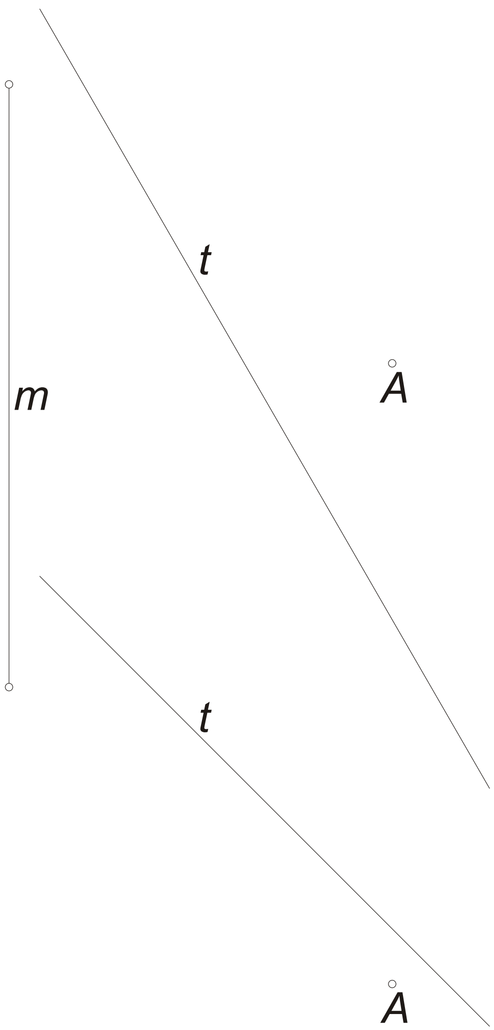
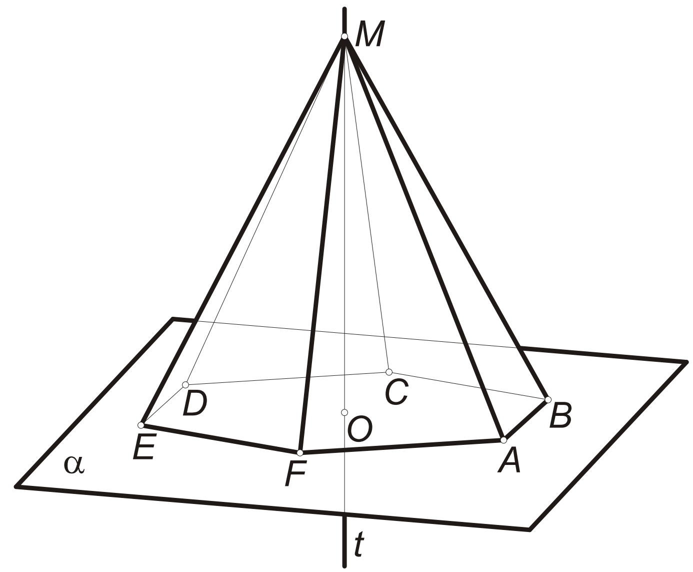
Metric constructions

Construction of a hexagon-based, regular pyramid with an oblique line as its axis

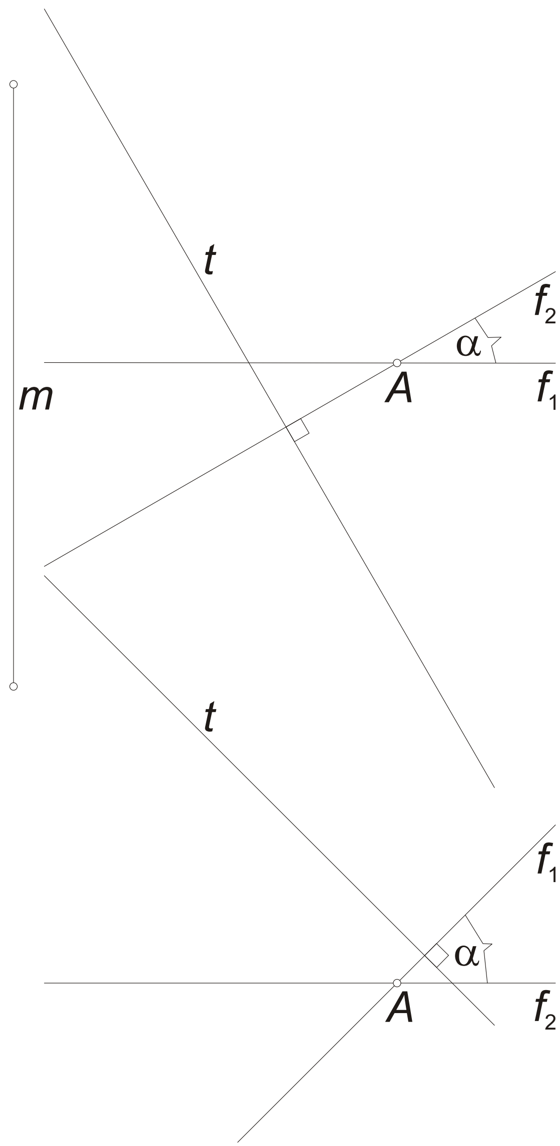


Exercise. *Given the axis t, the height m of a hexagon-based regular pyramid $ABCDEFM$, and also the vertex A of its base $ABCDEF$. Construct the pyramid. Show visibility assuming that the pyramid is made of plates, and that its base is removed.*

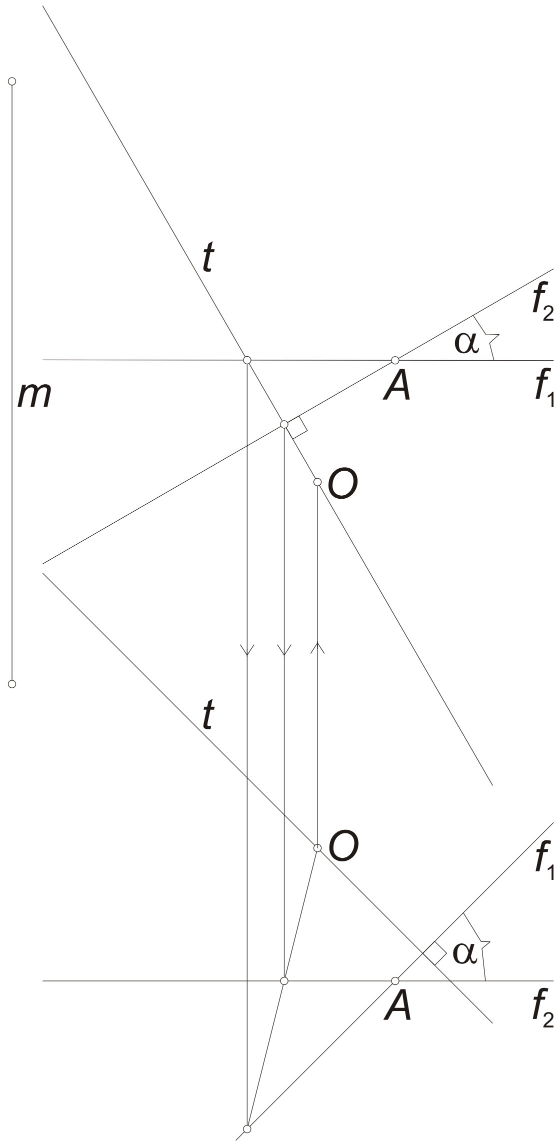


We carry out the construction in the following steps.

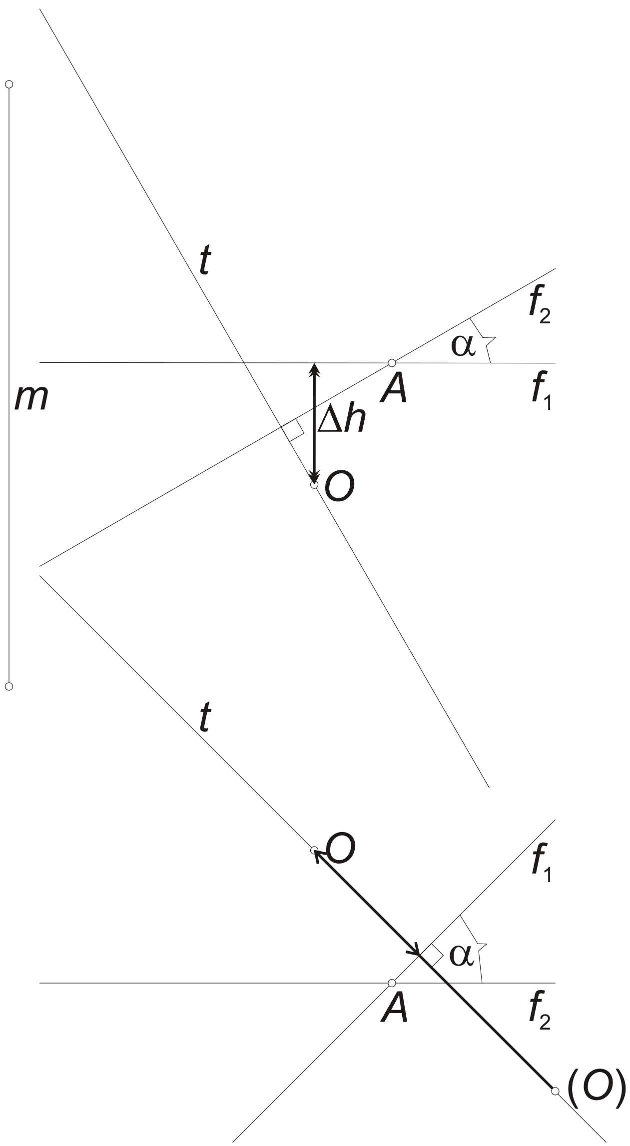
(1) Construction of the plane α of the base: $A \in \alpha \perp t$.
(2) Construction of the center O of the base: $O = \alpha \cap t$.
(3) a) Rotating α into horizontal position.
b) Construction of the base $ABCDEF$.
c) Rotating α back: constructing the two projections of the base.
(4) Measuring the height m from the center O on the axis t.



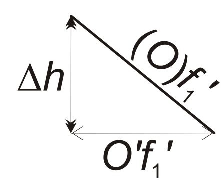
We draw the horizontal f_1 and the vertical f_2 lines of the plane of the base, passing through A. Since both are perpendicular to t, we have $A' \in f_1' \perp t'$, and $A'' \in f_2'' \perp t''$.

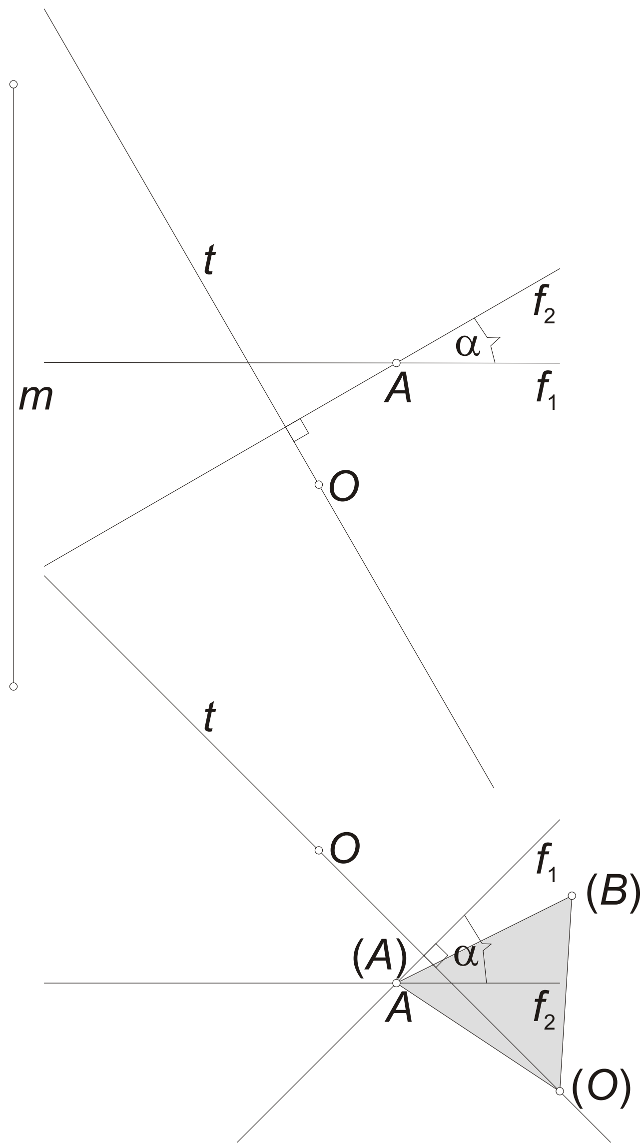


We construct the intersection point O of the axis t and the plane α.

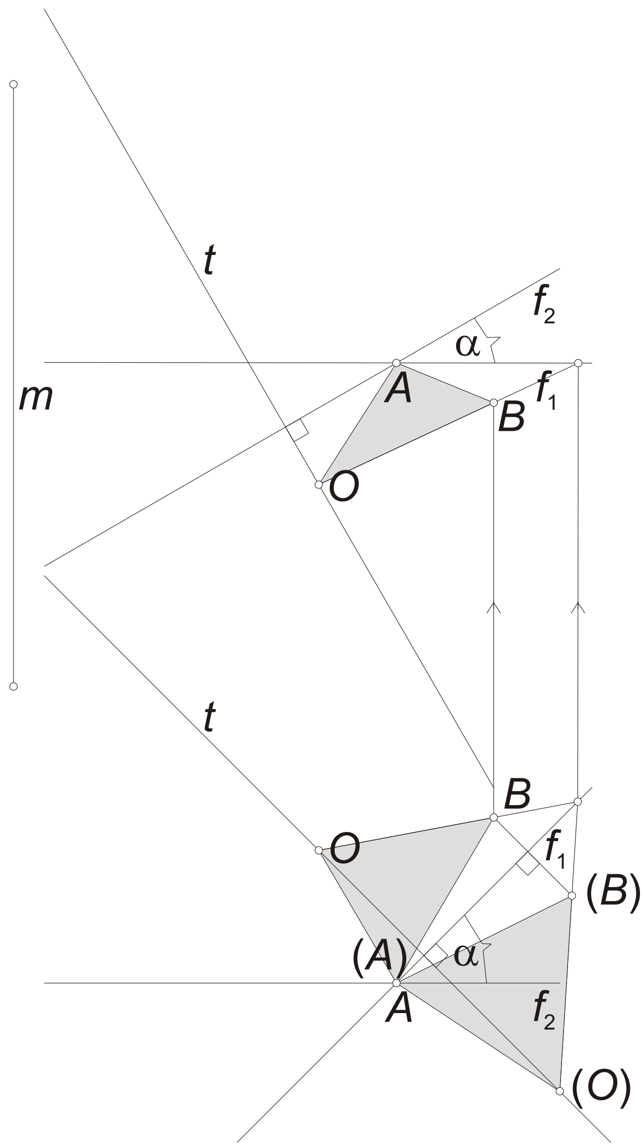


We rotate the plane α, about f_1 as its axis, into horizontal position.
Then we draw the difference triangle of O and f_1: the length of the horizontal leg is the distance of O' and f_1'; the length of the vertical leg is the difference of the levels of O'' and f_1''; and the hypotenuse shows the real distance of O and f_1. We need to measure this length from f_1' to obtain the rotated copy (O) of O.

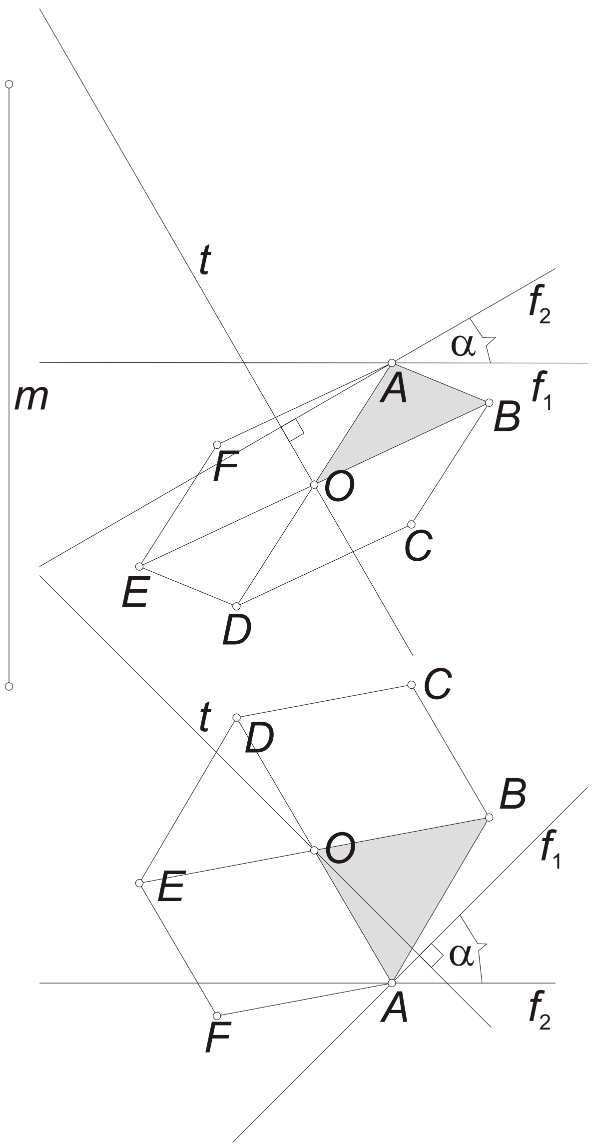




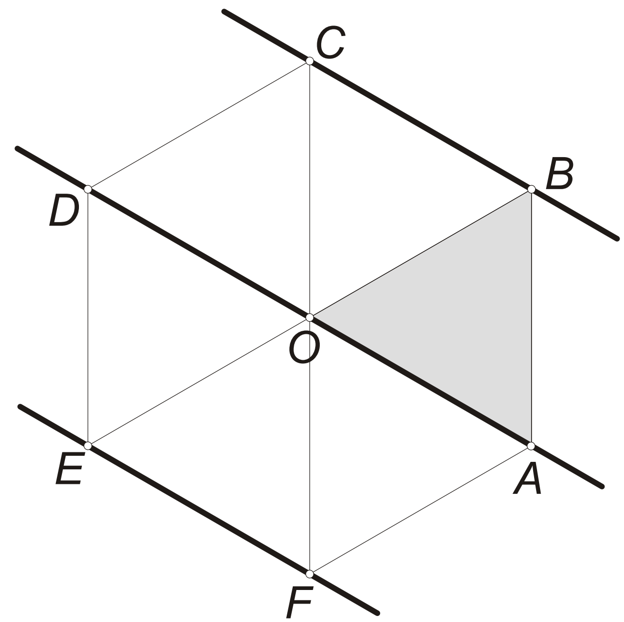
In the rotated plane we construct the hexagon $(A)(B)(C)(D)(E)(F)$. More specifically, we construct only its one sixth part: the regular triangle $(O)(A)(B)$. We will see that it is sufficient to construct the projections of the hexagon.



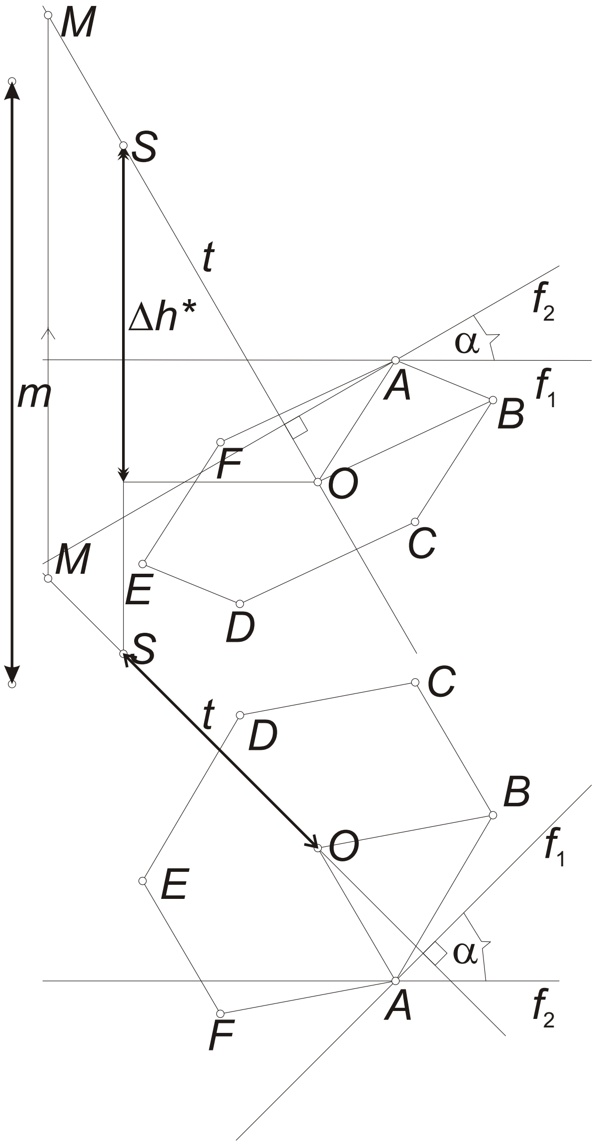
We rotate back the base of the plane, and determine the horizontal and the vertical projections of the triangle ABO.
We find the intersection point of $(O)(A)$ and f_1'. This point remains the same during the rotation, and thus, it coincides with the horizontal projection of the corresponding point. Using its line of recall, we can find the vertical projection of the point as well.
The horizontal projection of a point and its rotated copy can be connected by a line perpendicular to the axis f_1'. Thus, $B'(B) \perp f_1'$, from which, we can find B', and then, by its line of recall, B'' as well.



We construct the remaining vertices of the hexagon. We use the fact that in a regular hexagon, the opposite sides are parallel to the diagonal connecting the remaining two vertices and their length is half the length of this diagonal. These properties are preserved by a projection.



Thus, reflecting AB about O we obtain the projections of D and E. Then, drawing the lines, through B and E, respectively, parallel to the diagonal AD we obtain the sidelines containing BC and EF. Similarly, the lines through D and A, respectively, parallel to BE, contain the edges CD and FA.

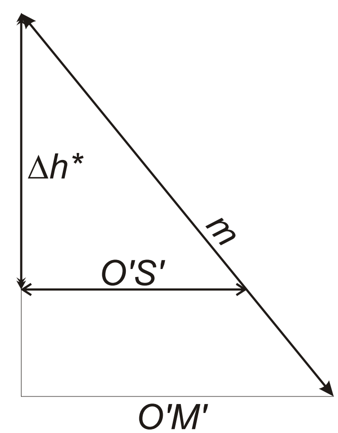


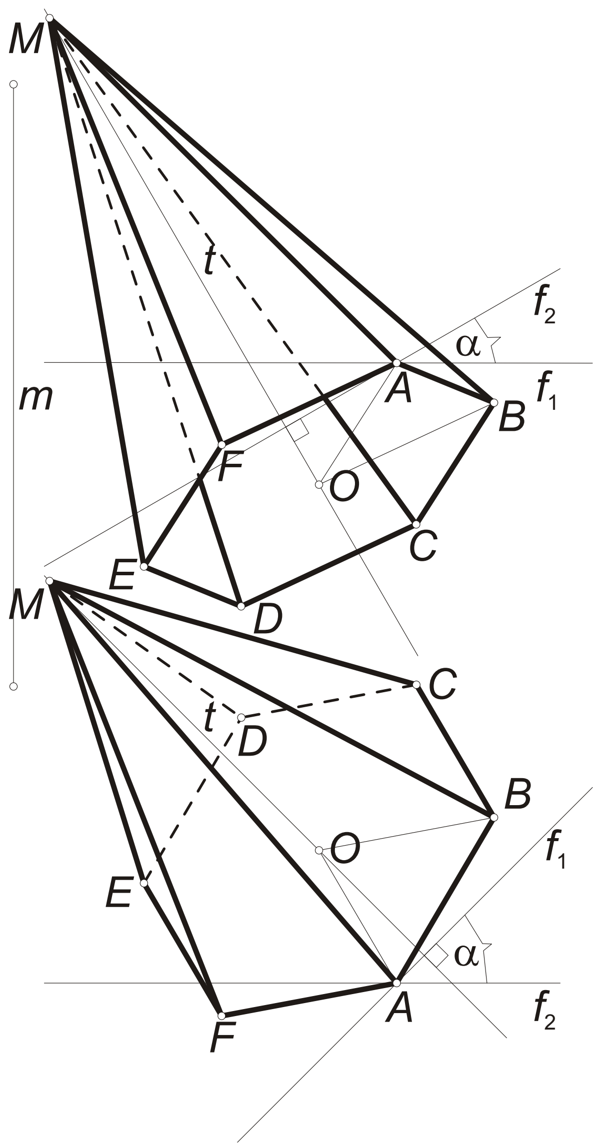
Now we measure the height m of the pyramid on the axis t from O.

On the axis we choose an arbitrary auxiliary point S (different from O), and draw the difference triangle of the segment OS. In this, the horizontal leg is the distance of O' and S', the vertical leg is the difference of the levels of O'' and S''.

From one of the endpoints, say the one on the vertical leg, we resize this triangle in such a way that the length of its hypotenuse is m.

The horizontal leg of the resized triangle is equal to the distance of O' and M'. This needs to be measured on t' from O' to draw M'. We can obtain M'' using the line of recall of the points.





Finally, we draw the lateral edges of the pyramid, and examine visibility according to the conditions listed in the Exercise.